FMPR 101—27.1
MAY 1966

THE ECONOMIC ORDER QUANTITY PRINCIPLE AND APPLICATIONS

A GSA HANDBOOK

GENERAL SERVICES ADMINISTRATION
WASHINGTON, D. C.

FSN 7610-00-543-6765

INTRODUCTION

This handbook is issued pursuant to the authority contained in FPMR 101-27, which also provides policies, principles, and guidelines to be used in the management of Government-owned inventories of personal property.

The Economic Order Quantity (EOQ) principle has been one of the most effective tools used in inventory management. Scientifically sound, this principle has found practical application in inventory control at all distribution levels.

In order to enhance its value to the many Federal agencies which have initiated EOQ procedures, this handbook provides comprehensive coverage of the EOQ principle and its applications. The handbook is divided into three chapters and two appendixes. Chapter 1 contains an explanation of the derivation of the EOQ principle and provides a guide for facilitating application of this principle. Chapter 2 explains the EOQ principle in depth — what it is; why it works; how it is modified; significance of errors; and the relationship of the Economic Purchase Quantity (EPQ) to EOQ. Chapter 3 describes methods of EOQ application — determining costs; finding EOQ by formulas, tables, and other devices; estimating EOQ effects; EOQ modifications; and finding the Economic Purchase Quantity (EPQ). Appendix A extracts and simplifies the methods in chapter 3 pertaining to cost analysis, constructing an EOQ table, determining EOQ effects and modifying the EOQ table. Appendix B is a table of square root values.

There are many methods for finding the Economic Order Quantity, all of which stem from the EOQ principle. Formulas may be used to describe the principle and each method, and by computing portions of the formulas in advance, they may be adapted for simpler use in tables, charts, nomographs, etc. While it is not necessary to use formulas when applying EOQ, they are helpful in understanding the applications. The formulas are concise descriptions of the computations necessary to find EOQ and, except for finding the square root, involve only simple arithmetic. Finding the square root is also simplified, however, by looking up square root values in tables such as appendix B of this handbook.

The handbook is identified under Federal stock number 7610-543-6765 in the GSA General Stores Stock Catalog and additional copies may be ordered in the same manner as other items shown in that catalog.

For sale by the Superintendent of Documents, U.S. Government Printing Office
Washington, D.C. 20402 - Price $1.50

Stock No. 022-000-00042-0

TABLE OF CONTENTS

CHAPTER 1. BASIC CONCEPT OF ECONOMIC ORDER QUANTITY

Paragraph Titles	Paragraph Numbers
General	1
Inventory Levels	2
Figure 1-2.1. Replenishment Cycle and Inventory Levels	
Figure 1-2.2. Net Replenishment Order Quantity	
Balancing Opposing Costs	3
Figure 1-3.1. Ordering and Holding Costs	
Figure 1-3.2. Months of Supply and Order Frequency	
Figure 1-3.3. The Economic Order Quantity	
Figure 1-3.4. The EOQ Formula	
Figure 1-3.5. EOQ Table	
Figure 1-3.6. Costs for EOQ and Fixed Operating Level	

CHAPTER 2. THE ECONOMIC ORDER QUANTITY PRINCIPLE OF STOCK REPLENISHMENT

Elements of Ordering and Holding Costs	1
Figure 2-1.1. Order Frequency and Ordering Costs	
Figure 2-1.2. Increased Cost per Order	
Figure 2-1.3. Inventory and Holding Costs	
Figure 2-1.4. Increased Holding Cost per Dollar of Inventory	
Figure 2-1.5. Increased Requirements Value	
Modifications of EOQ	2

Paragraph Titles | Paragraph Numbers

Figure 2-2.1. Modifications of EOQ

Figure 2-2.2. Range of EOQ Modifications

Figure 2-2.3. Orders and Inventory for Fixed Operating Levels

Effects of EOQ Errors 3

Figure 2-3.1. EOQ for Changing Cost Estimates

Figure 2-3.2. Effects of EOQ Errors

The Economic Purchase Quantity. 4

Figure 2-4.1. Economic Purchase Quantity

Figure 2-4.2. Total Costs for EOQ and EPQ

CHAPTER 3. METHODS FOR APPLICATION OF EOQ PRINCIPLE

Cost Analysis 1
Formulas for Finding EOQ 2

Figure 3-2.1. Use of Formula for EOQ in Units

Figure 3-2.2. Use of Formula for EOQ in Number of Orders per Year

Figure 3-2.3 Use of Formula for EOQ in Number of Months of Supply

EOQ Tables 3

Figure 3-3.1 Use of Formula for Finding Monthly Requirements Value for EOQ Expressed in Number of Months of Supply

Figure 3-3.2. Use of Formula for Finding Range of Monthly Requirements Values for EOQ Expressed in Number of Months of Supply

Paragraph Titles		Paragraph Numbers
Figure 3-3.3.	Use of Basic Values to Compute a 50 Cost Ratio Table	
Figure 3-3.4.	Use of Basic Values to Compute a 100 Cost Ratio Table	
Figure 3-3.5.	Errors in EOQ Table Values	
Figure 3-3.6.	EOQ Table in Number of Orders per Year	
Figure 3-3.7.	EOQ Table in Dollar Values	
Figure 3-3.8.	EOQ Tables in Unit Quantities	
Other EOQ Devices		4
Figure 3-4.1.	EOQ Chart in Unit Quantities	
Figure 3-4.2.	EOQ Nomograph in Unit Quantities	
Estimating EOQ Effects for Multi-Item Inventory		5
Figure 3-5.1.	Estimating Effects of EOQ	
Figure 3-5.2.	Estimating Effects of EOQ by Formula	
Modifications of EOQ		6
Figure 3-6.1.	Estimating Effects of Modified EOQ by Formula	
Figure 3-6.2.	Effects of Cost Factor Changes on Inventory and Orders	
Figure 3-6.3.	Finding Cost Factors for Modified EOQ	
Figure 3-6.4.	Effects of Cost Ratio Changes on Inventory and Orders	
Figure 3-6.5.	Relationship Between Cost Factor and Cost Ratio	
Figure 3-6.6.	Finding Cost Ratios for Modified EOQ	

Paragraph Titles | Paragraph Numbers

Figure 3-6.7. Effects of Changes in Requirements Values in EOQ Table on Inventory and Orders

Figure 3-6.8. Changing EOQ Table for Modified EOQ

Figure 3-6.9. Estimating Effects of Modified EOQ Table

The Economic Purchase Quantity 7

Figure 3-7.1. Evaluation of Economic Purchase Quantity (EPQ)

Figure 3-7.2. Expected Changes in Total Costs of Ordering and Holding for Changes from Basic EOQ

Figure 3-7.3. Estimating Total Costs for Basic EOQ

Figure 3-7.4. Economic Purchase Quantity Table

Figure 3-7.5. Evaluation of EPQ Limits

Figure 3-7.6. Computation of EPQ Limits

Appendix A. Instructions for Simplified EOQ Application

Appendix B. Square Root Values

CHAPTER 1. BASIC CONCEPT OF ECONOMIC ORDER QUANTITY

1. GENERAL. The "Economic Order Quantity"(EOQ) principle is a mathematically proven solution for arriving at the lowest total costs for ordering and holding inventory to meet expected supply requirements. Consequently, each civil executive agency is required, by FPMR 101-27.102, to follow this principle in the management of inventories. In its most concise form the economic order quantity principle can be stated in the formula: $Q = C\sqrt{Y}$. In words this formula says that the "economic order quantity" (Q) is the square root of the value of annual requirements (Y) times a cost factor (C) which includes the cost to order and the cost to hold stock.

2. INVENTORY LEVELS. The EOQ principle is an integral part of inventory management. Therefore, this paragraph 2 describes the inventory levels involved in inventory management and relates the EOQ principle to the appropriate level.

a. Leadtime, Operating, and Safety Levels. Inventory management generally encompasses the functions of determining future supply requirements and ordering and holding inventory to meet those requirements. As long as requirements continue to materialize for a particular item, the ordering and holding of inventory will be continued. This cyclic replenishment causes the inventory to be divided into three levels as shown in figure 1-2.1, below:

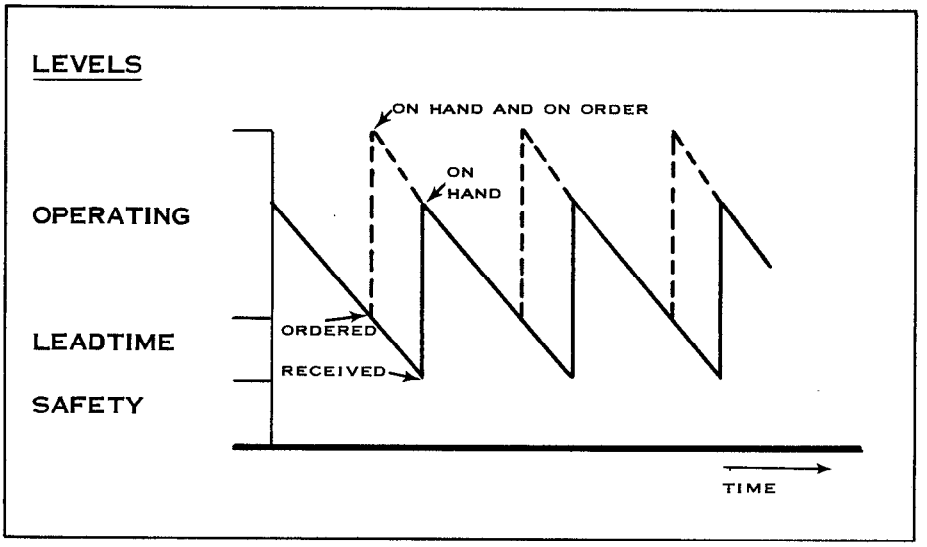

Figure 1-2.1 Replenishment Cycle and Inventory Levels

As indicated in figure 1-2.1 the operating level is expected to be issued between replenishment orders; the leadtime level is expected to be issued while the replenishment order is being prepared and delivery is being made; and the safety level is expected to be issued if the other levels are depleted.

b. EOQ and the Operating Level. The EOQ principle applies only to the operating level. Strictly speaking, it determines the economic quantity to order for the operating level when it is replenished. However, if the leadtime and safety levels are below their required quantities, at the time of ordering replenishment, the order quantity must be increased to restore those levels. Figure 1-2.2, below, illustrates the computation of an order quantity based on the stock required to replenish the leadtime and safety levels as well as the operating level.

LEVELS	QUANTITY REQUIRED
Operating	30
Leadtime	10
Safety	10
Total	50
Less Available Stock	- 5
Net Replenishment to Order	45

Figure 1-2.2. Net Replenishment Order Quantity

3. BALANCING OPPOSING COSTS. The EOQ principle is based on balancing the opposing costs of ordering and holding stock to the maximum extent practicable in order to obtain the minimum total of these costs. Figure 1-3.1, below, illustrates the opposing costs for ordering and holding stock and the interrelationship of such costs.

Operating Level (Units)	Months of Supply	Holding Costs	Order Frequency per Year	Ordering Costs	Total Costs
20	1	$ 1.00	12	$ 60.00	$ 61.00
40	2	2.00	6	30.00	32.00
60	3	3.00	4	20.00	23.00
80	4	4.00	3	15.00	19.00
120	6	6.00	2	10.00	16.00
240	12	12.00	1	5.00	17.00
480	24	$24.00	*	*	$ 26.50

*Note: The operating level of 24 months of supply would result in an order frequency of one order every other year. The ordering cost when averaged over a two year period would be $2.50 and the total average cost for holding and ordering over the two year period would be $26.50.

Figure 1-3.1. Ordering and Holding Costs

This table assumes that certain ordering costs are incurred each time an item is ordered and that certain holding costs are incurred for each unit held in stock. As the holding costs increase with the operating level, ordering costs decrease. Any reduction in one set of costs is opposed by an increase in the other. In order to find the most economical operating level the lowest total cost for ordering and holding must be found. In this case an operating level of 120, resulting in a total cost of $16.00, is the most economical operating level of those shown in figure 1-3.1.

a. Reason for Opposing Costs. The reason for these opposing costs is that holding costs are based on months of supply for the operating level which are opposed to the resulting order frequency, which is the basis for ordering costs. As shown in figure 1-3.1 holding costs increase with the months of supply and ordering costs increase with the order frequency per year but the order frequency decreases as the months of supply increase. In fact, doubling the months of supply in the operating level will cut the order frequency in half. This is known as an "inverse" or "reciprocal" relationship. Figure 1-3.2, below, illustrates the inverse relationship between months of supply and order frequency per year.

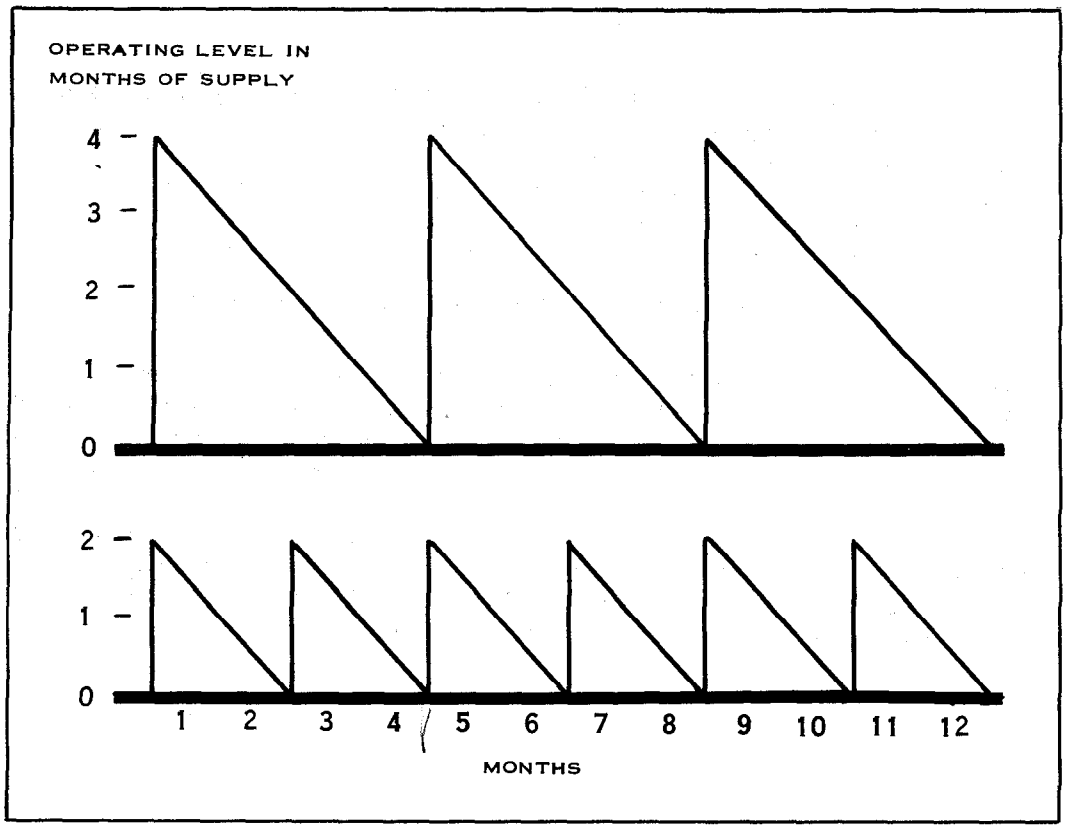

Figure 1-3.2. Months of Supply and Order Frequency

The upper chart shows how an operating level of four months of supply would have to be replenished three times a year (each peak representing a replenishment), while the lower chart cuts the operating level in half to two months of supply but in doing so must double the number of replenishments to six per year.

b. Lowest Total Cost. If cost data like that in figure 1-3.1 were plotted for every possible intervening operating level, figure 1-3.3, below, would be the result.

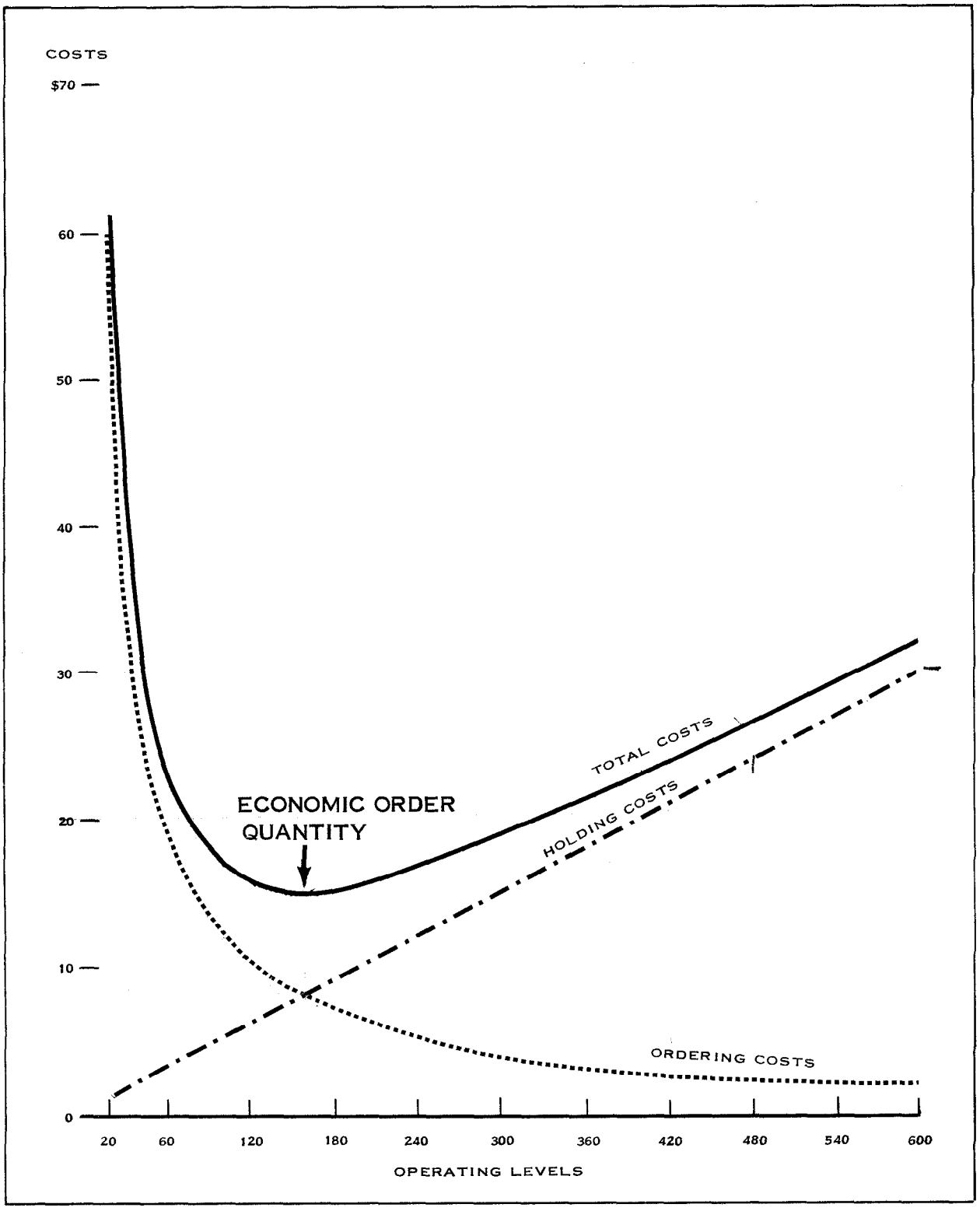

Figure 1-3.3. The Economic Order Quantity

In figure 1-3.3 it can be observed that as the operating level increases the holding costs increase while the ordering costs decrease. At the same time the total costs decrease to a low point after which they increase continuously. The operating level resulting in the lowest total cost is the economic order quantity. This operating level also corresponds with the point at which the holding costs and ordering costs are equal. In this case the economic order quantity is 155.

c. The EOQ Formula. Fortunately there is no need to go through all the work of computing the total costs for different operating levels as in figure 1-3.1 to arrive at the operating level with the lowest total cost. A general solution has been worked out which can find the economic order quantity directly. The formula which also expresses the EOQ principle is the general mathematical solution for finding the lowest point on the total cost line for any graph such as figure 1-3.3. Figure 1-3.4, below, gives the EOQ formula and illustrates its use in finding the economic order quantity.

Formula: $Q = C\sqrt{Y}$

Where Q = Value of Economic Order Quantity

Y = Value of Annual Requirement

C = Cost Factor based on $\sqrt{2\frac{P}{I}}$

P = Cost to Order an Item

I = Cost to Hold (as % of Average Inventory Value)

Illustration of Use where P = \$5*, I = 10%*, and Y = \$240

$Q = \sqrt{2 \times \frac{\$5}{.10}} \times \sqrt{\$240} = \sqrt{100} \times \sqrt{240} = 10 \times 15.5$

$Q = \$155$

*Note: The \$5 cost to order an item and 10% cost to hold used in this and most tables in this handbook, are used for illustration purposes only and should not be interpreted as a recommended standard for all inventories.

Figure 1-3.4. The EOQ Formula

d. Use of an EOQ Table. While the formula in figure 1-3.4 may be used as shown for determining the EOQ for each inventory item, there are methods for adapting the formula to more simplified use. One method is the use of an EOQ table, such as that shown in figure 1-3.5, below. The range of monthly requirement values for an item reflects the EOQ, expressed in months of supply, as indicated in figure 1-3.5. For example, an item with a monthly requirement value of $25 has an operating level of 6 months of supply with a value of $150 (6 x $25 = $150) according to this table.

Monthly Requirement*	Operating Level (Months of Supply)
$ 10 and less	12
10 to 20	9
20 to 40	6
40 to 60	5
60 to 100	4
100 to 160	3
160 to 240	2 $\frac{1}{2}$
240 to 400	2
400 to 800	1 $\frac{1}{2}$
800 and over	1

*Note: While it is technically correct, as in this and other tables shown, to have the same monthly requirements value end one range and begin the next, it may be more convenient for administering the EOQ table procedure, to separate each range by a $ .01 interval. For example, the ending values may be changed to $9.99, $19.99, $39.99, etc., or the beginning values may be changed starting with the second line to $10.01, $20.01, $40.01, etc.

Figure 1-3.5. EOQ Table

Figure 1-3.5 is based on cost estimates of $5 per order and a 10% rate to hold or any other combination of cost estimates resulting in an ordering to holding cost ratio of 50, (e.g., $5/.10 = 50, $10/.20 = 50, etc.). This specific table may be used for general and administrative items of stock requisitioned from Government sources, or procured under blanket purchase arrangements or from Federal Supply Schedule Contracts.

e. Fixed Operating Level vs. EOQ. It is principally because of the divergency in dollar value of requirements that the EOQ method can accomplish its savings. The EOQ method will reduce the total number of orders by ordering items with low dollar requirements infrequently, and by ordering smaller quantities of the items with high dollar requirements, the total inventory value is reduced. Figure 1-3.6, below, illustrates how uneconomical a single fixed operating level for both low and high value items would be.

	Monthly Requirement	Operating Level	Average Inventory	Holding Costs	Order Frequency	Ordering Costs	Total Cost
(3-Month Operating Level)	$ 10	$ 30	$ 15	$ 1.50	4	$20	$ 21.50
	1,000	3,000	1,500	150.00	4	20	170.00
							$191.50
(EOQ)	$ 10	$ 120	$ 60	$ 6.00	1	$ 5	$ 11.00
	1,000	1,000	500	50.00	12	60	110.00
							$121.00

Figure 1-3.6. Costs for EOQ and Fixed Operating Level

Assuming cost estimates of $5 to order and 10% to hold, a three-month operating level for an item, whose monthly requirement value is $10, would result in virtually twice the total cost of the 12-month (EOQ) operating level ($21.50 compared to $11.00). Using a three-month operating level, for an item with a monthly requirements value of $1,000 instead of the one-month operating

level prescribed by EOQ, results in over 50% additional costs ($170.00 compared to $110.00). By examining the costs, it is evident that the fixed three-month operating level causes excessive ordering for the item with low value requirements and excessive inventory for the item with high value requirements. As in the previous tables, the holding costs in figure 1-3.6 refer only to operating level. In computing total costs for a procedure actually in operation, holding costs for all inventory, including safety stock would have to be considered, unless some practical method for extracting data on operating stock were available.

CHAPTER 2. THE ECONOMIC ORDER QUANTITY PRINCIPLE OF STOCK REPLENISHMENT

1. ELEMENTS OF ORDERING AND HOLDING COSTS. Since changes in the operating level and order frequency affect the ordering costs and holding costs, an understanding of EOQ is facilitated if the cost elements affected by these changes are known.

a. Ordering Costs. Replenishment orders may involve costs for the following operations:

Reviewing the stock position of the item.
Preparing and processing the requisition or purchase request.
Selection of a supplier (includes preparing and issuing price inquiries and receiving, tabulating, and evaluating quotations).
Preparing and processing the purchase order.
Expediting the order.
Preparing and processing receiving reports.
Receiving, inspecting, and storing stock.
Posting receipts on stock record.
Preparing and processing payments.

b. Cost to Order an Item. In order to apply the basic EOQ principle an estimate should be made of the costs for all the ordering operations that would be involved for each additional replenishment order. This cost estimate must consider performance of the operations, wages, material, and equipment required. While actual ordering costs do vary for each item a single cost estimate is adequate for items involving substantially the same costs. For example, items ordered from GSA or purchased locally may have the same ordering costs but items procured through formal advertising are likely to have substantially higher ordering costs. The cost estimate for ordering an item is generally expressed as the cost per order so as to reflect how ordering costs will change with the order frequency. For example, the ordering costs in figure 2-1.1, below, are based on a cost estimate of $5 per order. As illustrated, ordering costs increase or decrease depending upon the ordering frequency. Thus one order per year costs $5, 2 orders cost $10, 3 orders cost $15, etc.

Order Frequency per Year	Ordering Costs
12	$60.00
6	30.00
4	20.00
3	15.00
2	10.00
1	5.00
$\frac{1}{2}$	2.50

Figure 2-1.1. Order Frequency and Ordering Costs

c. Increase in Cost Per Order. The ordering costs in figure 2-1.1 are the same as those shown in figure 1-3.1. Both are based on a cost estimate of $5 per order. If the cost per order were higher, it would tend to increase the operating level determined under the EOQ principle. In figure 2-1.2, below, the cost per order has been increased to $20. In comparing figure 2-1.2 with figure 1-3.1 it will be noted that the increased cost per order has changed the ordering costs and total costs so that the operating level that results in the lowest total cost is now 12 months of supply instead of 6.

Operating Level	Months of Supply	Holding Costs	Order Frequency per Year	Ordering Costs	Total Costs
20	1	$ 1.00	12	$ 240.00	$ 241.00
40	2	2.00	6	120.00	122.00
60	3	3.00	4	80.00	83.00
80	4	4.00	3	60.00	64.00
120	6	6.00	2	40.00	46.00
240	12	12.00	1	20.00	32.00
480	24	24.00	$\frac{1}{2}$	10.00	34.00

Figure 2-1.2. Increased Cost Per Order

d. Holding Costs. Holding stock may involve costs for the following operations or elements:

Taking physical inventory.
Preparing and processing inventory adjustments.
Prevention of deterioration.
Repacking and rewarehousing.
Storage space.
Interest on inventory investment.
Inventory losses for stock declared excess, obsolescence, deterioration, theft, and damage.

(1) Storage space costs need only be considered when they are affected by changes in the stock quantities being held.

(2) The interest on inventory investment reflects the cost of the use of the funds tied up in the inventory being held. Based on a study made by GSA, the rate of interest considered applicable to the Government's investment in inventory is 4 1/2% per year.

(3) Inventory losses due to excess, obsolescence, etc., are generally written off in the accounting records. An average annual rate, calculated from the losses in the accounting records over an extended period of time, should be used as an item of cost for holding inventory.

e. Holding Cost per Dollar of Inventory. An estimate of the cost involved to hold each additional unit of inventory over a period of time is needed to apply the EOQ principle. Since the interest on inventory investment and inventory losses usually account for the largest portion of the holding costs and are usually expressed as a percentage rate of the dollar value of the inventory, a cost estimate representing all elements of holding cost is often similarly expressed. Items with substantially the same holding costs should use the same estimate of holding cost per dollar of inventory. Separate cost estimates may be needed however for items subject to rapid obsolescence or for items which require servicing to prevent deterioration. The estimate of holding cost per dollar of inventory expressed as a percentage of the inventory value carried during the year will reflect how holding costs change with the inventory value. The holding costs in figure 2-1.3, below, are based on a cost estimate of 10% of the "average" dollar value of inventory carried during the year and a unit price of $1.00 for this item. The average dollar value of inventory, carried during the year is estimated as the dollar value of one-half the operating level. As illustrated, the holding costs increase as the number of months of supply in the operating level increase.

Operating Level	Months of Supply	Average Value of Inventory	Holding Costs
20	1	$ 10.00	$ 1.00
40	2	20.00	2.00
60	3	30.00	3.00
80	4	40.00	4.00
120	6	60.00	6.00
240	12	120.00	12.00
480	24	240.00	24.00

Figure 2-1.3. Inventory and Holding Costs

f. Increase in Holding Cost per Dollar of Inventory. The holding costs in figure 2-1.3 are the same as those shown in figure 1-3.1. Both are based on a cost estimate of 10% of the average dollar value of inventory. If the holding cost per dollar of inventory were higher, it would tend to decrease the operating level determined under the EOQ principle. In figure 2-1.4, below, the cost to hold has been increased to 40% of the average dollar value of inventory. In comparing figure 2-1.4 with figure 1-3.1 it will be noted that the increased holding cost per dollar of inventory has changed the holding costs and total costs so that the operating level that results in the lowest total cost is now 4 months of supply instead of 6 months.

Operating Level	Months of Supply	Average Inventory	Holding Costs	Order Frequency per Year	Ordering Costs	Total Costs
20	1	$ 10.00	$ 4.00	12	$60.00	$64.00
40	2	20.00	8.00	6	30.00	38.00
60	3	30.00	12.00	4	20.00	32.00
80	4	40.00	16.00	3	15.00	31.00
120	6	60.00	24.00	2	10.00	34.00
240	12	120.00	48.00	1	5.00	53.00
480	24	240.00	96.00	$\frac{1}{2}$	2.50	98.50

Figure 2-1.4. Increased Holding Cost per Dollar of Inventory

g. Increase in Requirements Dollar Value. Increases in the dollar value of requirements will also increase the holding costs and tend to decrease the operating level determined under the EOQ principle. In figure 2-1.5, below, the holding cost per dollar of inventory remains 10% and the cost per order remains $5 but the quantity required for a one-month supply has been increased to 80 units thereby increasing the average dollar value of inventory for each operating level. In comparing figure 2-1.5 with figure 1-3.1 it will be noted that the increased average dollar value of inventory has changed the holding costs and total costs so that the operating level that results in the lowest total cost is now 4 months of supply instead of 6 months. Thus, the effect of an increased requirements value is similar to the effect of an increased holding cost per dollar of inventory. Figure 2-1.5 also illustrates the need for having a reasonably good estimate of requirements in order to apply the EOQ principle. The demand for many items are subject to wide fluctuation or to factors of seasonal or trend variations which must be considered in order to obtain a good estimate of requirements. The determination of requirements is important for all inventory management decisions.

Operating Level	Months Of Supply	Average Inventory	Holding Costs	Order Frequency per Year	Ordering Costs	Total Costs
80	1	$ 40.00	$ 4.00	12	$ 60.00	$ 64.00
160	2	80.00	8.00	6	30.00	38.00
240	3	120.00	12.00	4	20.00	32.00
320	4	160.00	16.00	3	15.00	31.00
480	6	240.00	24.00	2	10.00	34.00
960	12	480.00	48.00	1	5.00	53.00
1,920	24	960.00	96.00	$\frac{1}{2}$	2.50	98.50

Figure 2-1.5. Increased Requirements Values

(1) It should be noted that EOQ is affected by the "dollar" value of requirements. Thus the increased holding costs and resulting decrease in operating level under EOQ in figure 2-1.5 could have resulted from a price increase and without any change in the unit quantity required.

(2) The cost per order and the holding cost per dollar of inventory do not vary to any substantial degree and are used repetitively for given groups of items. On the other hand, requirements must be determined separately for each item on a relatively frequent basis.

2. MODIFICATIONS OF EOQ. The EOQ principle prescribes operating levels in accordance with the requirements value of each item. The total number of orders or inventory expected to result however may exceed the organization's immediate capacity to process orders, or its available investment funds or storage space. Until such capacity restrictions can be eliminated, the EOQ principle may be modified so as to obtain operating levels that will result in the lowest total cost within the capacity limitation.

a. Constant Change Factor. The basic EOQ principle may be modified by increasing or decreasing operating levels by a constant factor that will restrict either the total number of orders or the total inventory investment within the desired limits. Figure 2-2.1 below illustrates basic EOQ and the modifications of EOQ designed to limit inventory or to limit the number of orders as might be applicable to two items carried in stock. The basic EOQ in section A results in the lowest total cost, but if the average inventory of $522 were considered too high or the order frequency

of 9 1/3 were considered excessive, modifications would be necessary. Section B reduces the average inventory by one-third ($522 to $348), by reducing the operating level for each item by one-third. Section C reduces the number of orders by one-fourth (9 1/3 to 7), by increasing the operating level for each item by one-third. (As discussed earlier operating level has an inverse or reciprocal relationship to order frequency, therefore to obtain 3/4 of the order frequency, a change factor of 4/3 is used on each operating level.) While only basic EOQ (i.e., economic order quantity based on the best cost estimates for ordering an item and holding inventory) will result in the lowest total cost, a modification of EOQ which applies a constant change factor to the operating level of each item will result in the lowest cost within the inventory or order frequency limitation. Chapter 3 describes many simple methods for modifying EOQ.

	Monthly Requirement	Months Supply	Operating Level	Average Inventory	Holding Costs	Number of Orders	Order Costs	Total Costs
A. (Basic EOQ)	$ 16	9	$ 144	$ 72	$ 7.20	1 1/3	$ 6.66	$ 13.86
	600	1 1/2	900	450	45.00	8	40.00	85.00
			$1,044	$522	$52.20	9 1/3	$46.66	$ 98.86
B. (Modified EOQ to Limit Inventory)	$ 16	6	$ 96	$ 48	$ 4.80	2	$10.00	$ 14.80
	600	1	600	300	30.00	12	60.00	90.00
			$ 696	$348	$34.80	14	$70.00	$104.80
C. (Modified EOQ to Limit Number of Orders)	$ 16	12	$ 192	$ 96	$ 9.60	1	$ 5.00	$ 14.60
	600	2	1,200	600	60.00	6	30.00	90.00
			$1,392	$696	$69.60	7	$35.00	$104.60

Figure 2-2.1. Modifications of EOQ

b. Combinations of Inventory and Orders. The range of EOQ modifications is so great generally, that the use of any other ordering system for reasons of funds, manpower, or space limitation is unwarranted and wasteful.

(1) The curve in figure 2-2.2, below, shows a range of combinations of inventory and order frequency that are possible modifications of EOQ for the two items illustrated in figure 2-2.1. Point A on the curve corresponds to the basic EOQ in section A of figure 2-2.1 and is plotted against 9 1/3 on the order frequency scale and against $522 on the average inventory value scale. Any other point on the curve would represent a modified EOQ obtained by changing the operating level for each item by a constant factor. Point B represents the modification in section B and point C represents the modification in section C of figure 2-2.1.

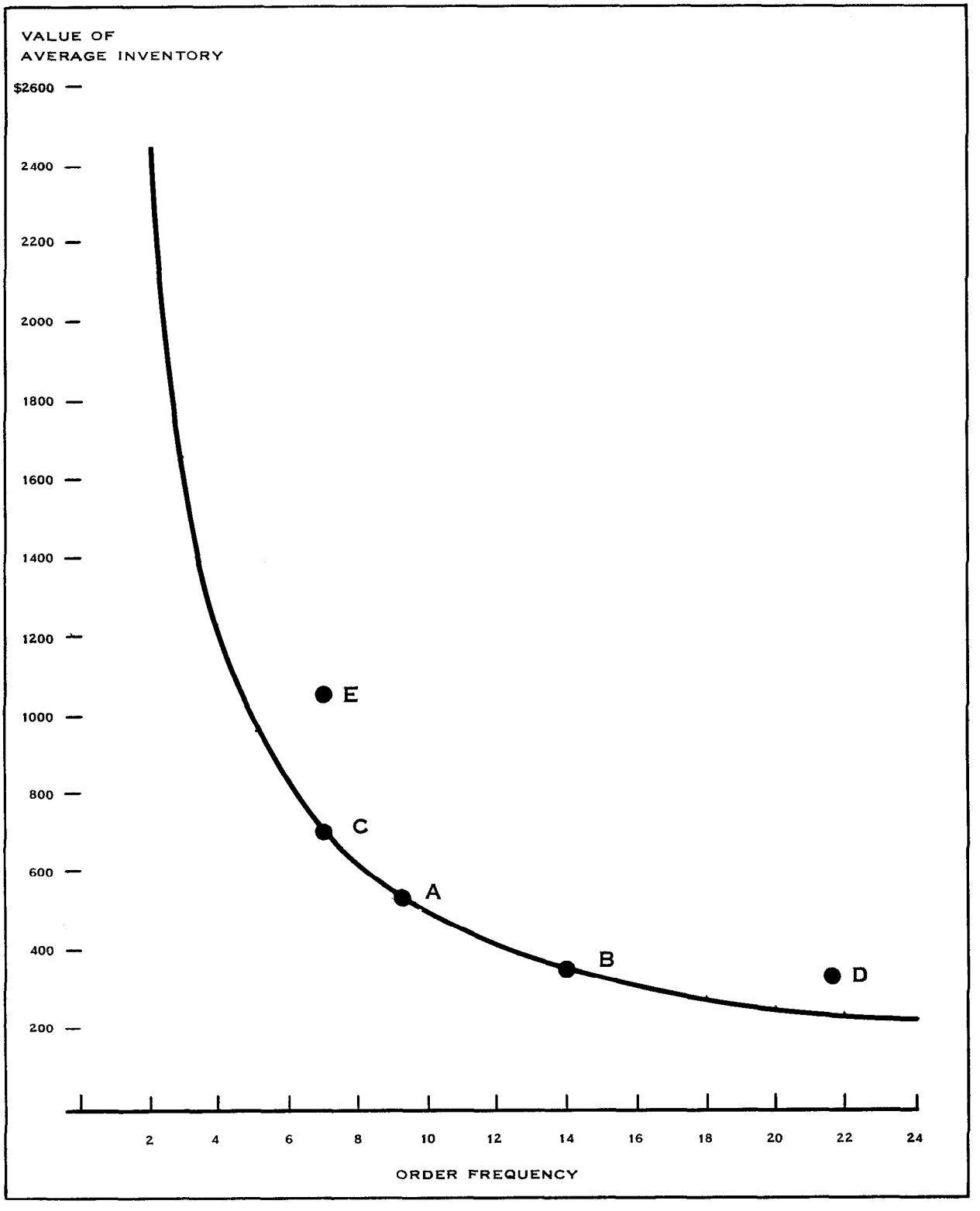

Figure 2-2.2. Range of EOQ Modifications

(2) Combinations of order frequency and average inventory not on the curve of figure 2-2.2 are neither basic EOQ nor modifications of EOQ. For example, point D represents 21.8 orders and a $339 average inventory value resulting from a fixed 1.1-month operating level for each item and point E represents 7 orders and $1,047 average inventory resulting from a fixed 3.4-month operating level for each item. While point D (use of a fixed 1.1-month operating level) successfully limits the average inventory to $339, about the same as the $348 for the modified EOQ of point B, the resulting total cost as shown in figure 2-2.3, below, is about 40% higher because of the higher order frequency. Similarly point E (use of a fixed 3.4-month operating level) maintains the same order frequency of 7 orders as the modified EOQ of point C, but the resulting total cost is about 35% higher because of the larger inventory.

Monthly Requirement	Months of Supply	Operating Level	Average Inventory	Holding Costs	Order Frequency	Order Costs	Total Costs
$ 16	1.1	$ 18	$ 9	$ .90	10.9	$ 55.00	$ 55.90
600	1.1	660	330	33.00	10.9	55.00	88.00
			$ 339	$33.90	21.8	$110.00	$143.90
$ 16	3.4	$ 54	$ 27	$ 2.70	3.5	$ 17.50	$ 20.20
600	3.4	2,040	1,020	102.00	3.5	17.50	119.50
			$1,047	$104.70	7.0	$ 35.00	$139.70

Figure 2-2.3. Orders and Inventory for Fixed Operating Levels

3. EFFECTS OF EOQ ERRORS. Whenever any substantial change is indicated or suspected in the cost per order, the holding cost per dollar of inventory, or requirements value estimate, a review and recomputation of that estimate is necessary. Despite periodic review, however, actual costs and requirements values will still vary from their estimates. Therefore, it is important to know the effect of errors for these estimates.

a. Error in Cost to Order. The modifications of EOQ illustrated in figure 2-2.1 may also be used to illustrate the effect of EOQ errors. If the cost to order an item were erroneously estimated at $2.50 instead of $5 then the operating levels in section B of figure 2-2.1 would be considered as the basic EOQ.

Figure 2-3.1 below computes the total costs of sections A and B of figure 2-2.1 when the cost per order is estimated as $2.50 instead of $5.00. The effects of using the incorrect cost estimate of $2.50 per order instead of the correct estimate of $5.00 can be measured from the data in sections A and B of figure 2-2.1. The total of the operating levels would be $696 instead of $1,044 and the total costs would be $104.80 instead of $98.86. Therefore, by using a cost estimate per order which is 50% lower than it should be, the total operating levels would be about 30% lower than it should be and the total costs would be only about 5% greater than they should be.

	Monthly Requirements	Months Supply	Operating Level	Average Inventory	Holding Costs	Order Frequency	Order Costs	Total Costs
A.	$ 16	9	$ 144	$ 72	$ 7.20	1 1/3	$ 3.33	$10.53
	600	1 1/2	900	450	45.00	8	20.00	65.00
			$1,044	$522	$52.20	9 1/3	$23.33	$75.53
B.	$ 16	6	$ 96	$ 48	$ 4.80	2	$ 5.00	$ 9.80
(Basic	600	1	600	300	30.00	12	30.00	60.00
EOQ)			$ 696	$348	$34.80	14	$35.00	$69.80

Figure 2-3.1. EOQ for Changing Cost Estimates

b. Net Error. The effects on EOQ and total costs of any size error in cost to order, cost to hold or in requirements values, can be similarly computed. Figure 2-3.2, below, shows the expected effects on EOQ and total costs of using EOQ methods based on cost estimates or requirements values with various percentages of error. This figure indicates that a net error in the costs and requirements estimates may be as much as 75% too low or 300% too high and still result in only a 25% increase in total cost.

Net % Error of Cost Estimates and Requirements Value	Resulting % Change in EOQ	Resulting % Change in Total Costs
- 75	- 50	+ 25
- 50	- 29	+ 6
- 35	- 19	+ 2
- 25	- 13	+ 1
00	00	00
+ 25	+ 12	+ 1
+ 50	+ 22	+ 2
+ 100	+ 41	+ 6
+ 200	+ 73	+ 15
+ 300	+ 100	+ 25

Figure 2-3.2. Effects of EOQ Errors

(1) Figure 2-3.2 may be used to determine the effects of the net error in the cost estimates and requirements values in using EOQ for one or more inventory items.

(2) The net error of the costs and requirements value estimates is computed as:

$$\text{Net Error} = \frac{p\,y}{i}$$

where p = % error in cost per order

i = % error in holding cost per dollar of inventory

y = % error in requirements value

The net effect of these errors may sometimes be cumulative but more frequently they tend to cancel each other.

c. Error Tolerance. Despite the fact that a large error in estimating costs and requirements values will have a relatively minor effect on the total ordering and holding costs, there are other effects of these errors which must be considered besides the total ordering and holding costs, when determining how accurate these estimates should be. Large errors in these estimates could result in overstocking and costly excess losses, or understocking and costly shortages. The seriousness of these effects for a particular item or class of items should also be considered in determining the accuracy required for these estimates. Since, under EOQ, large errors in requirements value estimates will have a relatively small effect on the total holding and ordering costs, convenient EOQ tables such as shown in figure 1-3.5 can be devised so as to assign a single EOQ quantity to any item whose requirement value falls in a given range of requirement values. Consequently despite these errors, the EOQ table when properly devised, offers a simple EOQ guide with a built-in tolerance which will assure that increased holding and ordering costs resulting from its use will be negligible.

4. THE ECONOMIC PURCHASE QUANTITY. While the EOQ principle determines the operating level which will result in the lowest total cost for ordering and holding, it assumes a fixed unit purchase price for the item. However, suppliers sometimes offer price discounts for quantity purchases or similarly, carriers offer reduced transportation rates for given quantities. Since the stipulated minimum quantities which must be ordered under the discount terms may differ considerably from the economic order quantity determined on the basis of no discount, a broader evaluation of the economic order quantity based on the discount terms must be made to determine the appropriate order quantity. The determination of the purchase quantity based on this broader evaluation is known as the Economic Purchase Quantity (EPQ). Methods for determining EPQ are described in chapter 3.

a. EOQ vs EPQ. Figure 2-4.1, below, illustrates the broader evaluation of EOQ when quantity discounts are offered. The curve representing total holding and ordering costs is the same as the total cost curve of figure 1-3.3 which was based on the data in figure 1-3.1. In this illustration, while the economic order quantity is 155, a 1% discount is offered if a minimum quantity of 240 is purchased or a 2% discount if a minimum quantity of 360 is purchased. A comparison of EOQ and the alternative quantity discounts indicates that a 1% discount for ordering 240 units would reduce the total costs below the EOQ total costs while a 2% discount for ordering 360 units would not reduce total costs below EOQ costs.

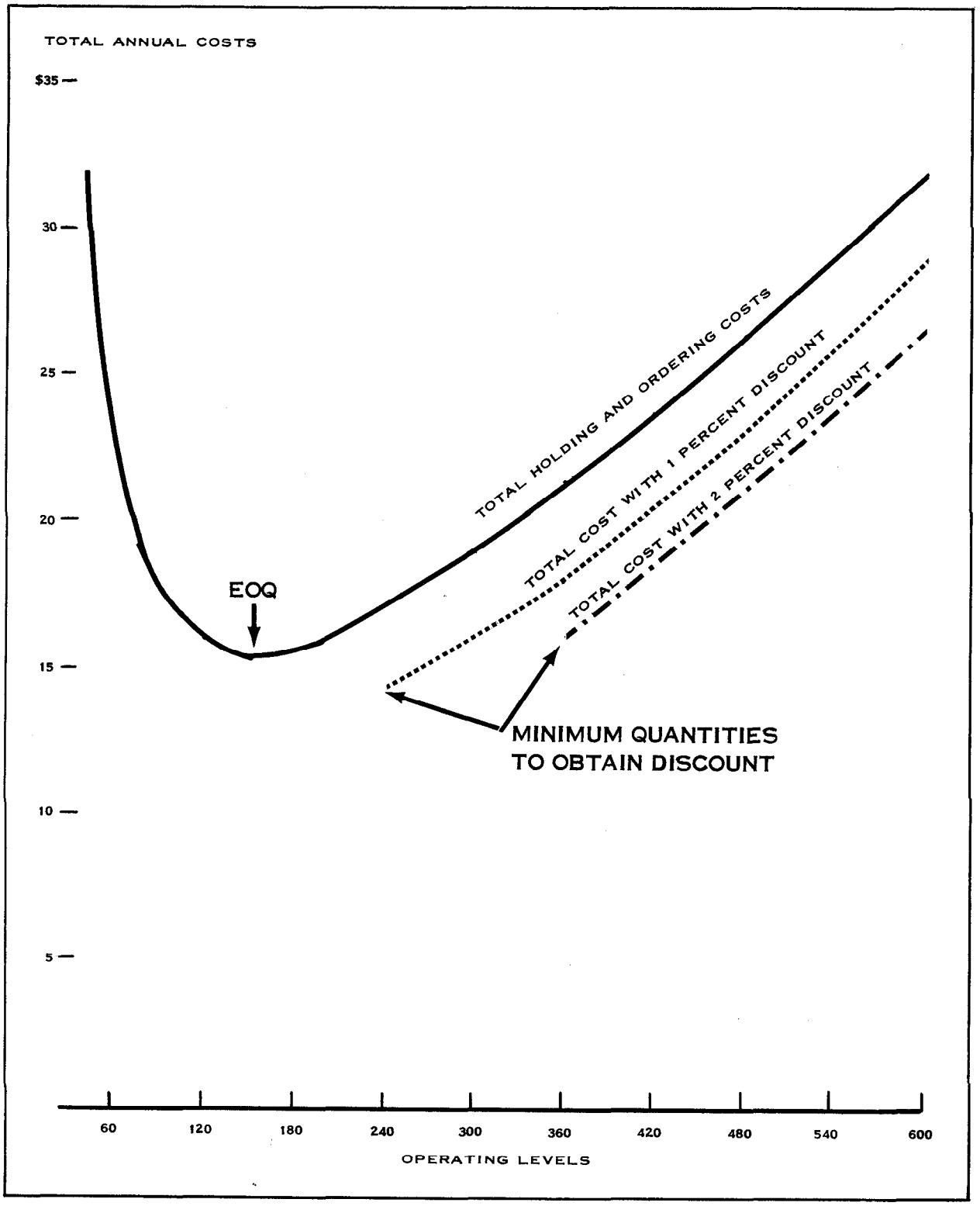

Figure 2-4.1. Economic Purchase Quantity

b. Overall Cost Evaluation. Figure 2-4.2, below, shows the detailed cost development for the EOQ and for the minimum order quantities when price discounts of 1% and 2% are offered. The holding costs for the increased average inventories are based on the discounted unit price (i.e., 120 units at $.99 = $118.80 and 180 units at $.98 = $176.40). The discount savings are computed as the percentage discount of the annual requirements value, (i.e., 1% of $240 = $2.40, and 2% of $240 = $4.80). In order to obtain the discount saving, EOQ must be sacrificed. However, when the increased total of ordering and holding costs offsets the discount saving, there is no benefit in purchasing the discount quantity.

	EOQ	1% Discount	2% Discount
Annual Requirement	240	240	240
Operating Level	155	240	360
Average Inventory	77.5	120	180
Unit Price	$ 1.00	$ .99	$ .98
Average Inventory Value	$ 77.50	$118.80	$176.40
Holding Costs	$ 7.75	$ 11.88	$ 17.64
Order Frequency	1.55	1	2/3
Ordering Costs	$ 7.75	$ 5.00	$ 3.33
Total Costs	$ 15.50	$ 16.88	$ 20.97
Discount Saving	0	2.40	4.80
Net Total Cost	$ 15.50	$ 14.48	$ 16.17

Figure 2-4.2 Total Costs for EOQ and EPQ

CHAPTER 3. METHODS FOR APPLICATION OF EOQ PRINCIPLE

1. COST ANALYSIS. There are two main elements inherent in the EOQ principle. One is the value of requirements and the other is the cost factor. The cost factor is based on the costs involved for each additional replenishment order and for each additional dollar value of stock being held. Some estimate of these costs must be made before the basic EOQ can be obtained. In order to estimate the costs for ordering and holding stock, a study of the appropriate cost elements for recent supply operations is necessary. The effort and expense of obtaining accurate cost estimates should depend in large part upon the importance and value of the items which are involved. Generally the initial implementation of EOQ need not wait for the development of more accurate cost estimates. Even when based on poor cost estimates EOQ is likely to be more economical than any other system. Any necessary refinements in records and methods of estimating costs may be made at subsequent cost review periods.

a. Cost Elements. Cost estimates for ordering and holding are needed for each group of items that is expected to have substantially the same costs. These estimates are usually based on data of the past year's supply operations. Only those operations which pertain to ordering and holding inventory to meet future demands are involved. For each cost item shown in paragraphs 1 a and 1 d in chapter 2 any significant costs involving labor, materials and supplies, equipment and repair which are likely to change with the frequency of ordering or the quantity ordered, are developed and tallied. The method of extracting the data would vary according to the availability of accounting, cost, and management records. These costs usually reflect the activities of other elements within an organization besides the supply operation. Accounting and data processing usually involve an important share of these costs.

b. Costs to Order and to Hold. The ordering and holding costs computed as shown in this paragraph 1 b will serve as estimates of the costs to order and to hold stock in conformance with the EOQ principle.

(1) The cost per order for a group of items is obtained by dividing the total costs for ordering those items over a period of time (usually one year) by the number of times the items were ordered during that period. For example, if the total annual costs for ordering a group of items amounted

to $25,000 and these items were ordered a total of 5,000 times during the year, the cost per order would be:

$$\frac{\$25,000}{5,000} = \$5$$

(2) The cost to hold stock for a group of items is obtained by dividing the total costs for holding stock for those items over a period of time (usually one year) by the value of the average inventory (both operating and safety stock) being held during that period. For example, if the total annual costs for holding stock for a group of items amounted to $10,000 and the value of the average inventory being held during the year was $100,000, the holding cost per dollar of inventory would be:

$$\frac{\$\ 10,000}{\$100,000} = \$.10 \text{ (expressed as a percentage } 100 \text{ X } \$.10 = 10\%)$$

(3) Since the inventory may vary considerably during the course of one year, it is necessary to use the average of several stock balances recorded during the year in computing the holding cost per dollar of inventory. There is no need to differentiate between operating and safety stock since the cost to hold should be the same for both.

2. FORMULAS FOR FINDING EOQ. Many methods are available to find EOQ, however all methods are derived from the EOQ formula $Q = C\sqrt{Y}$ as explained in chap. 1-1 of this handbook. In this par. 2 the accepted formula methods for finding EOQ are described.

a. EOQ in Dollar Value. The formula $Q = C\sqrt{Y}$ is a method of finding EOQ in dollars. Figure 1-3.4 defines the elements of the EOQ formula and illustrates its computational use for a $5 cost estimate for ordering, a 10% cost estimate for holding stock, and an annual requirement value of $240. Square root values may be obtained from appendix B.

b. EOQ in Unit Quantity. The quantity in units could be obtained by dividing the EOQ dollar value by the unit price. If the unit price were $.50 then EOQ in units would be:

$$\frac{\$155}{\$.50} = 310$$

However by dividing the EOQ formula in figure 1-3.4 by unit price, it could be used to obtain EOQ directly in units. The formula for obtaining EOQ directly in units and its use is illustrated in figure 3-2.1.

Formula: $Q_u = C\sqrt{\frac{Y_u}{V}}$

Where Q_u = Economic Order Quantity in Units

Y_u = Annual Requirement in Units

V = Unit Price

C = Cost Factor based on $\sqrt{\frac{2P}{I}}$

Use of Formula where $Y_u = 480$, $V = \$.50$, and $C = \sqrt{\frac{2 \times 5}{.10}} = 10$

$$Q_u = 10\sqrt{\frac{480}{.50}} = 10\sqrt{960} = 10 \times 31$$

$$Q_u = 310$$

Figure 3-2.1. Use of Formula for EOQ in Units

c. Requirements Data Increments. The formulas of figure 1-3.4 and 3-2.1 can be used for requirements data expressed in monthly and quarterly as well as annual increments. The only thing to remember in using requirements data of less than one year is to adjust the cost for holding inventory accordingly. If the rate for holding stock is 10% per year, it must be divided by 4 for quarterly requirements data and divided by 12 for monthly requirements data. The illustration of the use of the formula in figure 1-3.4 then would be:

For quarterly requirements data:

$$Q = \sqrt{2 \times \frac{\$5}{.10/4}} \times \sqrt{60} = \sqrt{400} \times \sqrt{60} = 20 \times 7.75$$

$$Q = \$155$$

For monthly requirements data:

$$Q = \sqrt{2 \times \frac{\$5}{.10/12}} \times \sqrt{20} = \sqrt{1,200} \times \sqrt{20} = 34.64 \times 4.47$$

$$Q = \$155$$

d. EOQ in Orders per Year. The need may also arise for expressing EOQ in terms of number of orders per year and number of months of supply. In order to obtain the number of orders per year it is necessary to determine how many multiples of the order quantity make up the annual requirement. For example, an order quantity of $40 goes into the annual requirement of $240 six times and therefore would result in six orders per year. An order quantity of $155 would result in 1.55 orders per year $\left(\frac{240}{155} = 1.55.\right)$ The EOQ formula in figure 1-3.4 may also be adapted to express EOQ directly in number of orders per year by dividing it into the annual requirements value. The resulting formula and an illustration of its use is shown in figure 3-2.2, below.

Formula: $F = \sqrt{\frac{Y}{C}}$

Where F = Number of Orders per Year

Y = Value of Annual Requirement

C = Cost Factor based on $\sqrt{2\frac{P}{I}}$

Use of Formula where P = $5, I = 10%, and Y = 240

$$F = \frac{\sqrt{240}}{\sqrt{2 \times \frac{\$5}{.10}}} = \frac{\sqrt{240}}{\sqrt{100}} = \frac{15.5}{10}$$

$$F = 1.55$$

Figure 3-2.2. Use of Formula for EOQ in Number of Orders per Year.

e. EOQ in Months of Supply. To obtain the order quantity in months of supply it is necessary to determine what fraction the order quantity is of the annual requirement (the reciprocal of the number of orders per year) and convert this fraction of a year to months. For example, an order quantity of $40 makes up 1/6 of an annual requirement of $240, and 1/6 of a year is two months. Similarly an order quantity of $155 is 65% of the annual requirement of $240 and 65% of 12 months is 7.75 months. The formula in figure 3-2.2 may be adapted to express EOQ directly in number of months of supply by taking its reciprocal and converting it to monthly data. The resulting formula and an illustration of its use is shown in figure 3-2.3.

Formula: $N = \frac{C}{\sqrt{M}}$

Where N = Number of months of supply

M = Value of Monthly Requirement

C = Cost Factor based on $\sqrt{2\frac{P}{I}}$

Use of Formula where P = $5, I = 10% and M = $20

$$N = \frac{\sqrt{2 \times \frac{\$5}{.10/12}}}{\sqrt{20}} = \frac{\sqrt{1,200}}{\sqrt{20}} = \frac{34.64}{4.47}$$

N = 7.75

Figure 3-2.3. Use of Formula for EOQ in Number of Months of Supply

f. Limitations. When the requirements values are very high or very low, the EOQ formulas will result in order quantities that are either impractical or beyond the organization's ordering authority. Therefore by policy the order quantities resulting from the formulas will be restricted by upper and lower limits. For example, the upper and lower limits may be 12 months and one month of supply, respectively, in which case any computed

order quantities greater than 12 months of supply would be restricted to 12 months, and order quantities less than one month of supply would be restricted to one month. A similar restriction would be placed on individual items having limited shelf life.

3. EOQ TABLES. The EOQ formula and its various adaptations may be computed separately for each item or may be used to compute tables with EOQ values for various ranges of requirements values. EOQ tables are generally identified by their cost ratios (cost to order divided by annual rate to hold) or by the cost factor used in the EOQ formula, which is the square root of 2 times the cost ratio $\left(\sqrt{2\frac{P}{I}}\right)$ EOQ tables may only be used for items having the same cost ratio or cost factor as the table.

a. Requirements Range Formula. The chief advantage of an EOQ table is its simplicity of application. All the pertinent EOQ computations have been made beforehand and conveniently listed so that EOQ values for items with given ranges of requirements can be easily found. For example, the EOQ for an item whose cost ratio is 50 and monthly requirement is $90 could be found in figure 1-3.5 by looking for the range of monthly requirements which includes $90 and the EOQ in months of supply is given on that same line. The appropriate requirements range would be $60 to $100 and the EOQ for that line is four months of supply.

(1) The formula in figure 3-2.3 can be adapted to compute directly the monthly requirement value for any given EOQ value expressed in months of supply selected for an EOQ table. The adapted formula and an illustration of its use are shown in figure 3-3.1, below.

Formula: $M = \frac{24K}{N^2}$

Where M = Value of Monthly Requirement

N = Number of Months of Supply

K = Cost Ratio based on $\frac{P}{I}$

Where P = Cost per order

I = Annual rate to hold

Use of Formula where P = \$5, I = 10%, and N = 4

$$M = \frac{24 \times \frac{\$5}{.10}}{(4)^2} = \frac{24 \times 50}{4 \times 4} = \frac{1,200}{16}$$

M = \$75

Figure 3-3.1. Use of Formula for Finding Monthly Requirements Value for EOQ Expressed in Number of Months of Supply

(2) The computation in figure 3-3.1 indicates that an item with a monthly requirements value of \$75 and costs of \$5 to order and 10% to hold, has an EOQ of four months of supply. This was also shown in figure 1-3.5. However, that table shows a range of \$60 to \$100 with an applicable EOQ of four months of supply. In order to find the range of monthly requirements values for given EOQ months of supply an additional adaptation of the formula in figure 3-3.1 is necessary. The adapted formula and an illustration of its use are shown in figure 3-3.2, below. The illustration finds the range of monthly requirements values applicable to an EOQ of four months of supply by computing the beginning and ending values of the range. The beginning range value of \$60 is the smallest requirements value applicable to an EOQ of four months of supply and the largest requirements value applicable to an EOQ of five months of supply. Similarly the ending range value of \$100 is the largest requirements value applicable to an EOQ of four months of supply and the smallest requirements value applicable to three months of supply. By using the formula as shown

in figure 3-3.2 all the monthly requirements ranges shown in figure 1-3.5 may be obtained. It should be noted that the first two ranges in that table actually should be $11.11 and less, and $11.11 to $22.22; but they have been rounded for convenience.

Formula: $M = \frac{24K}{N_a N_b}$

Where M = Value of Monthly Requirement

N_a = Selected Number of Months of Supply on EOQ Table

N_b = Next Selected Number of Months of Supply on EOQ Table

K = Cost Ratio based on $\frac{P}{I}$

Use of Formula where $P = \$5$, $I = 10\%$, $N_a = 5$, $N_b = 4$

$$M = \frac{24 \times \frac{\$5}{.10}}{5 \times 4} = \frac{24 \times 50}{20} = 1.20 \times 50$$

$M = \$60$

Where $N_a = 4$, $N_b = 3$

$$M = \frac{24 \times \frac{\$5}{.10}}{4 \times 3} = \frac{24 \times 50}{12} = 2.00 \times 50$$

$M = \$100$

Figure 3-3.2. Use of Formula for Finding Range of Monthly Requirements Values for EOQ Expressed in Number of Months of Supply

b. Use of Basic Values. When the same EOQ values expressed in months of supply are to be used in preparing EOQ tables for various cost ratios, computational steps may be saved by using so-called "basic values" which have already computed everything else in the formula in figure 3-3.2 except the multiplication by the cost ratio (K). For example, the basic values 1.20 and 2.00 are in the last computational steps illustrating the use of the formula in figure 3-3.2.

(1) Figure 3-3.3, below, illustrates the use of basic values in computing the 50 cost ratio table of figure 1-3.5. The ranges of monthly requirements values are obtained simply by multiplying the cost ratio by the basic value ranges appropriate for each EOQ value. For example, by multiplying the cost ratio of 50 by the basic value range "1.20 to 2.00." which is appropriate for an EOQ of four months of supply, the monthly requirements range of $60 to $100 is obtained. In figure 3-3.3, the basic values shown in the first column are only applicable to the EOQ values, expressed in months of supply, in the last column. These same basic values may be used to compute any EOQ table, whatever the cost ratio may be, as long as the corresponding EOQ table values are acceptable.

Basic Values	Monthly Requirements (50 X Basic Values)	EOQ in Months of Supply
.20 and Less	$ 10 and Less	12
.20 to .40	10 to $ 20	9
.40 to .80	20 to 40	6
.80 to 1.20	40 to 60	5
1.20 to 2.00	60 to 100	4
2.00 to 3.20	100 to 160	3
3.20 to 4.80	160 to 240	2.5
4.80 to 8.00	240 to 400	2
8.00 to 16.00	400 to 800	1.5
16.00 and Over	800 and Over	1

Figure 3-3.3. Use of Basic Values to Compute a 50 Cost Ratio Table

(2) Figure 3-3.4, below, illustrates the computation of a 100 cost ratio table using the same basic values as in figure 3-3.3. The only difference in the computations of these tables is that the basic values are multiplied by 50 in figure 3-3.3 and 100 in figure 3-3.4.

Basic Values	Monthly Requirements (100 X Basic Values)	EOQ in Months of Supply
.20 and Less	$ 20 and Less	12
.20 to .40	20 to $ 40	9
.40 to .80	40 to 80	6
.80 to 1.20	80 to 120	5
1.20 to 2.00	120 to 200	4
2.00 to 3.20	200 to 320	3
3.20 to 4.80	320 to 480	2.5
4.80 to 8.00	480 to 800	2
8.00 to 16.00	800 to 1,600	1.5
16.00 and Over	1,600 and Over	1

Figure 3-3.4 Use of Basic Values to Compute a 100 Cost Ratio Table

c. Table Error. Although an EOQ table is simple to use it is not as accurate as the application of the EOQ formula for each item. While figure 1-3.5 indicates four months of supply for an item with a monthly requirement of $90, the EOQ as computed by formula is actually 3.65 months of supply. This error of + 10% (4/3.65 = 1.10) is considered tolerable since it will increase the total costs for ordering and holding by less than 1%. The effect of an EOQ error on total costs can be estimated from the last two columns of figure 2-3.2. The reason for these errors is the conciseness of the table. A table could be constructed to include 3.65 months of supply and other intermediate EOQ values, however the table then would be no longer simple and concise. Moreover it could never hope to include all the possible requirements values that might be needed.

(1) An important step in constructing an EOQ table, or in selecting one that has already been prepared, is to insure that the error of its EOQ values is within the necessary tolerance for the inventory items using it. Starting with the upper and lower limits of order quantity which are practical or allowable by policy, the selection of the intervening EOQ table values can increase or decrease the table's accuracy. The EOQ values in figure 1-3.5 for example, will never be greater than 23.0% above or 18.4% below the EOQ value computed by formula except for those items limited by policy to no less than one month or no more than 12 months of supply. As indicated in figure 2-3.2 errors of this size in the order quantity would increase the total cost for ordering and holding stock only by about 2%.

(2) Figure 3-3.5, below, illustrates how the errors for the EOQ table values are computed. The largest errors occur when an EOQ table value of 1.5 months of supply is used instead of a formula computed 1.22 months of supply and when six months of supply is used instead of 7.35 months of supply. The values 1.22 and 7.35, as well as all the other values shown in the second column are called "breakpoints." A breakpoint is the smallest EOQ value that can be represented by a given EOQ table value and the largest EOQ value that can be represented by the next successive EOQ table value. It corresponds to the beginning or ending range value of requirements for two successive EOQ table values. For example, 1.22 is the lowest EOQ value that could be represented by an EOQ table value of 1.5 months of supply and it also is the largest EOQ value that could be represented by an EOQ table value of one month of supply. The breakpoint between any two successive EOQ table values may be computed by taking the square root of their product ($\sqrt{N_a N_b}$). For example, the breakpoint between six and nine months of supply is $\sqrt{9 \times 6} = 7.35$.

EOQ Table Months of Supply	Breakpoints	Overestimates of Breakpoints		Underestimates of Breakpoints	
		Fraction	%	Fraction	%
12					
	10.39	12/10.39	+15.5	9/10.39	-13.4
9					
	7.35	9/ 7.35	+22.4	6/ 7.35	-18.4
6					
	5.48	6/ 5.48	+ 9.5	5/ 5.48	- 8.8
5					
	4.47	5/ 4.47	+11.8	4/ 4.47	-10.5
4					
	3.46	4/ 3.46	+15.6	3/ 3.46	-13.3
3					
	2.74	3/ 2.74	+ 9.5	2.5/ 2.74	- 8.8
2.5					
	2.24	2.5/ 2.25	+11.1	2/ 2.24	-11.2
2					
	1.73	2/ 1.73	+15.6	1.5/ 1.73	-13.3
1.5					
	1.22	1.5/ 1.22	+23.0	1/ 1.22	-18.0
1					

Figure 3-3.5. Errors in EOQ Table Values

d. EOQ in Orders per Year. The EOQ values in a table may be expressed in many ways besides months of supply. The formulas in figures 1-3.4, 3-2.1, and 3-2.2 may be used to compute tables with EOQ values expressed in dollars, units, and number of orders per year, respectively. A table expressing EOQ in number of orders per year would be very similar to a table expressing EOQ in months of supply. Figure 3-3.6 is a cost ratio 50 table with EOQ values expressed in number of orders per year. This table is not as useful as figure 1-3.5 in indicating the amount to order, but is useful in determining expected amount of order processing for each item ordered.

Monthly Requirements	EOQ in number of Orders per Year
$ 10 and Less	1
10 to $ 20	1.3
20 to 40	2
40 to 60	2.4
60 to 100	3
100 to 160	4
160 to 240	4.8
240 to 400	6
400 to 800	8
800 and Over	12

Figure 3-3.6. EOQ Table in Number of Orders per Year

e. EOQ in Dollar Value. It may be more useful to have a table with EOQ values expressed in dollars rather than in months of supply or number of orders per year. Figure 3-3.7, below, is a cost ratio 50 table with EOQ expressed in dollar values. There are notable differences between figure 1-3.5 and figure 3-3.7. First, the errors in using the EOQ values are different. While figure 3-3.7 provides a fixed EOQ dollar value for each requirements range, the EOQ expressed in months of supply is variable. On the other hand figure 1-3.5 provides a fixed EOQ in months of supply and a variable EOQ dollar value for each requirements range. For example, the fixed EOQ of six months of supply in figure 1-3.5 for the requirements range of $20 to $40, could vary from $120 to $240, while a fixed EOQ of $200 for the same requirements range could vary from five to ten months of supply in figure 3-3.7. The error in using either table, however, is about the same. The second difference is that the upper and

lower limits are expressed differently. While figure 1-3.5 limits the order quantities to no more than 12 months of supply and no less than one month of supply, figure 3-3.7 has a maximum order value of $1200 and a minimum order value of $100. The values of figure 3-3.7 may be extended to increase these limits or to show EOQ dollar values appropriately extended within the policy for stock limitation. For example, the minimum order value may be extended to $50 with "$4 and less" as its applicable range of monthly requirements or all EOQ values below $100 may be limited to 12 months of supply and appropriately extended EOQ values given in the table.

Monthly Requirements	EOQ in Dollar Value
$ 10 and Less	$ 100
10 to $ 20	135
20 to 40	200
40 to 60	240
60 to 100	300
100 to 160	400
160 to 240	480
240 to 400	600
400 to 800	800
800 and Over	1,200

Figure 3-3.7. EOQ Table in Dollar Values

f. EOQ in Unit Quantity. While a table expressing EOQ in dollar value may be somewhat more useful than a table with EOQ values in months of supply, additional computations are still required

to determine the order quantity in units. Since stock records are usually kept in units it would seem most desirable to have a table expressing monthly requirements and EOQ in units. Unfortunately tables giving EOQ in units lose their conciseness because of the vast array of unit prices applicable to each order quantity expressed in units. Figure 3-3.8, below, shows cost ratio 50 EOQ tables in unit quantities for various unit prices. These by no means exhaust the unit prices and EOQ tables that may be needed for ordering stock. While the number of tables may be consolidated by using tables for ranges of unit prices (thereby introducing another source of table EOQ error), the number of tables would still be imposing. Therefore, unless the volume of ordering warrants it, the use of EOQ tables expressed in unit quantities is not a satisfactory method for simplifying the computation of EOQ. Figure 3-3.8 is similar to figure 3-3.7 in that each requirements value is represented by a fixed EOQ dollar value and that each table has a maximum order value of $1200 and a minimum order value of $100. As with figure 3-3.7, the limits of figure 3-3.8 may be extended or converted to equivalent months of supply in line with policy limitations.

Unit Price $.50 Mo. Unit Reqmts.	EOQ	Unit Price $.25 Mo. Unit Reqmts.	EOQ	Unit Price $.10 Mo. Unit Reqmts.	EOQ	Unit Price $.05 Mo. Unit Reqmts.	EOQ
0 - 20	200	0 - 40	400	0 - 100	1,000	0 - 200	2,000
20 - 40	270	40 - 80	540	100 - 200	1,350	200 - 400	2,700
40 - 80	400	80 - 160	800	200 - 400	2,000	400 - 800	4,000
80 - 120	480	160 - 240	960	400 - 600	2,400	800 - 1,200	4,800
120 - 200	600	240 - 400	1,200	600 - 1,000	3,000	1,200 - 2,000	6,000
200 - 320	800	400 - 640	1,600	1,000 - 1,600	4,000	2,000 - 3,200	8,000
320 - 480	720	640 - 960	1,920	1,600 - 2,400	4,800	3,200 - 4,800	9,600
480 - 800	1,200	960 - 1,600	2,400	2,400 - 4,000	6,000	4,800 - 8,000	12,000
800 - 1,600	1,600	1,600 - 3,200	3,200	4,000 - 8,000	8,000	8,000 - 16,000	16,000
1,600 & over	2,400	3,200 & over	4,800	8,000 & over	12,000	16,000 & over	24,000

Unit Price $2.00		Unit Price $4.00		Unit Price $8.00		Unit Price $16.00	
0 - 5	50	0 - 2.5	25	0 - 1.25	13	0 - .625	7
5 - 10	68	2.5- 5	34	1.25- 2.5	17	.625- 1.25	9
10 - 20	100	5 - 10	50	2.5 - 5	25	1.25 - 2.5	13
20 - 30	120	10 - 15	60	5 - 7.5	30	2.5 - 3.75	15
30 - 50	150	15 - 25	75	7.5 - 12.5	38	3.75 - 6.25	19
50 - 80	200	25 - 40	100	12.5 - 20	50	6.25 -10	25
80 - 120	240	40 - 60	120	20 - 30	60	10 -15	30
120 - 200	300	60 - 100	150	30 - 50	75	15 -25	38
200 - 400	400	100 - 200	200	50 - 100	100	25 -50	50
400 & over	600	200 & over	300	100 & over	150	50 & over	75

Figure 3-3.8. EOQ Tables in Unit Quantities

4. OTHER EOQ DEVICES. Other devices besides tables and formulas are used to compute EOQ. These devices usually take the form of charts, nomographs, and slide rules. All are based, nevertheless, on the EOQ formulas.

a. EOQ Chart. An EOQ chart consists of a series of plotted EOQ values which were computed for given requirements values.

(1) Figure 3-4.1, below, illustrates an EOQ chart for a 50 cost ratio expressed in unit quantities. Each diagonal line represents a different unit price and the point at which it intersects a given requirements value on the vertical scale will indicate the EOQ value measured on the horizontal scale. For example, the point where the $.10 unit price line crosses the monthly requirements line of 480 units would be the same point at which it crosses the EOQ line of 2,400 units. Similarly, the $4.00 unit price line indicates an EOQ for 60 units where the monthly requirement is 12.

(2) Figure 3-4.1 is more concise and simpler to compute than figure 3-3.8 since only two points must be computed to draw each unit price line. However, the values are more difficult to read, especially the logarithmic scales used in figure 3-4.1. While an evenly spaced arithmetic scale would be easier to read, all applicable values could not be shown on one page and the unit prices would be represented by curves thereby requiring many points to be computed in order to plot each curve. Charts expressing EOQ in dollar value or months of supply may also be computed. These would show only one diagonal line for a given cost ratio or cost factor.

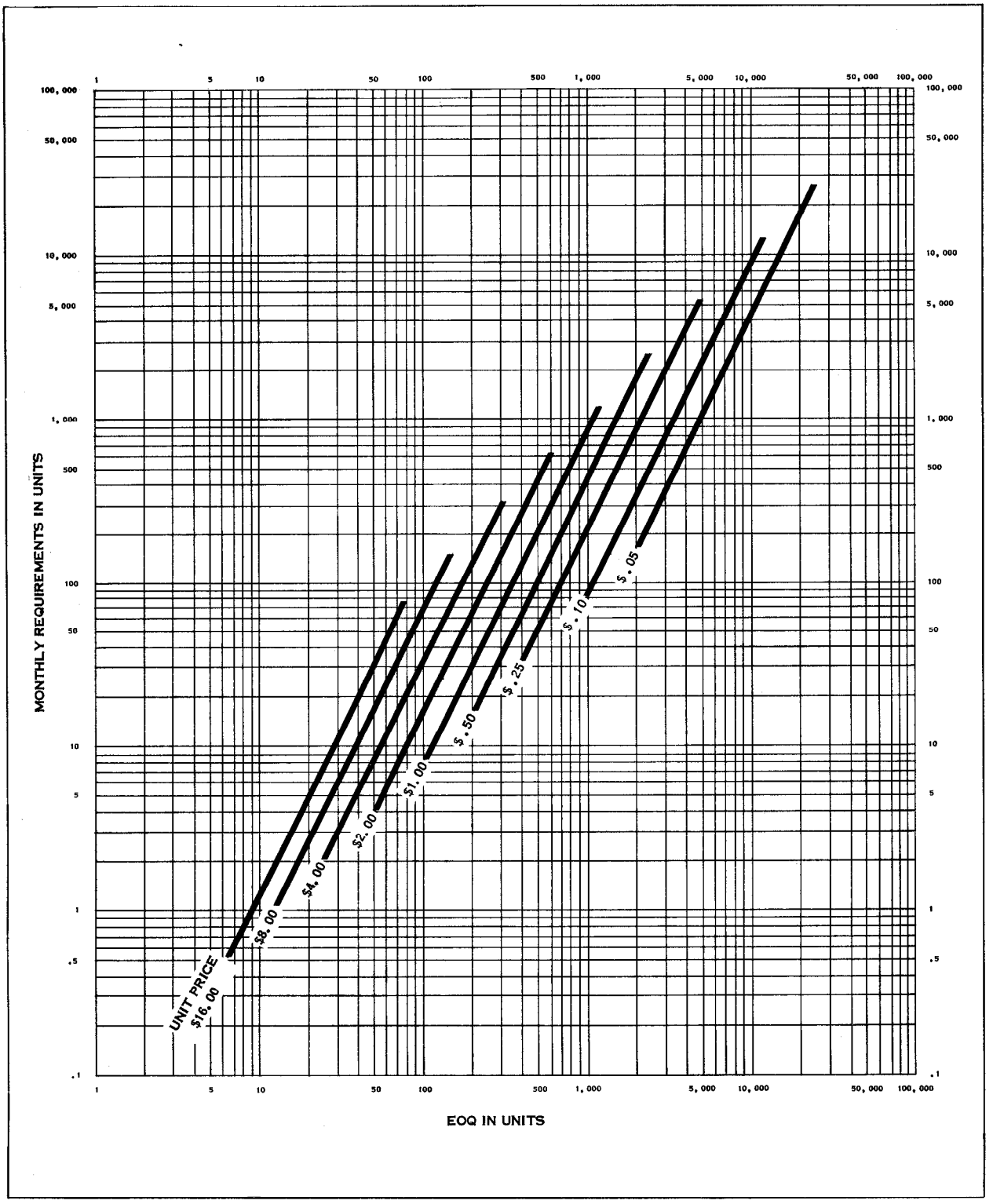

Figure 3-4.1. EOQ Chart in Unit Quantities (Cost Ratio 50)

b. EOQ Nomographs. Nomographs and slide rules are computational devices that multiply and divide by lining up the appropriate numbers which are marked off on logarithmic scales. The computation on a nomograph is accomplished by lining up the factors by means of a straight edge while a slide rule lines up the factors by moving the numerical scales. Figure 3-4.2, below, illustrates an EOQ nomograph expressed in unit quantities. It can compute the same solution as figure 3-4.1. For example, a straight edge crossing 480 units on the monthly requirements scale and $.10 on the unit price scale, would cross the EOQ scale at 2,400 units. As in the case of the EOQ chart, logarithmic scales on the EOQ nomograph may be difficult to read.

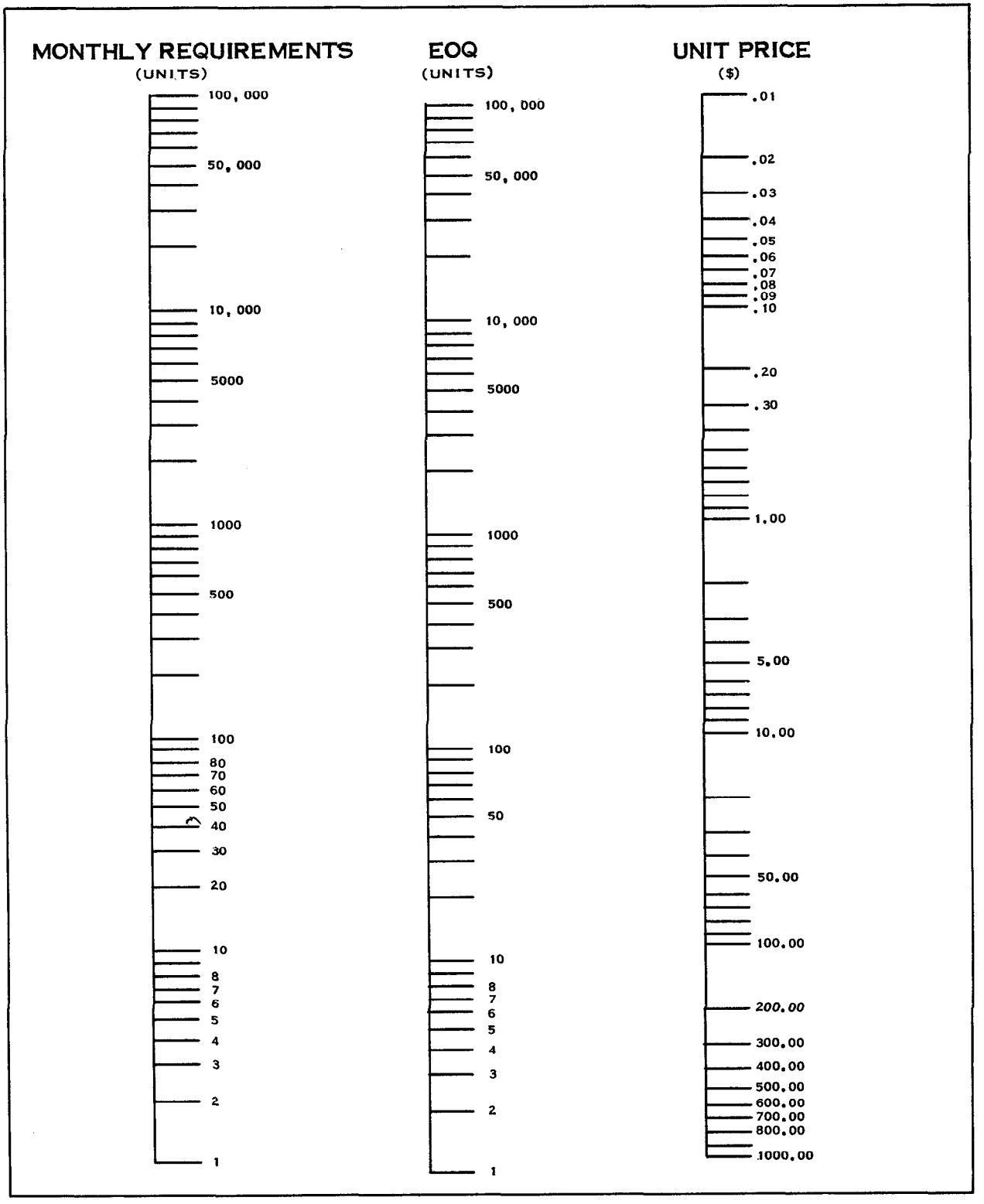

Figure 3-4.2. EOQ Nomograph in Unit Quantities

ESTIMATING EOQ EFFECTS FOR MULTI-ITEM INVENTORY. When a given EOQ formula, table, or other device is used, the order quantity and order frequency are determined by the requirements value of each item. Therefore, it is possible to project these data at any given time to find the expected average inventory investment (excluding safety stock*) and the expected number of orders per year for all items ordered in accordance with the procedure.

a. Estimating by Requirements Ranges. Figure 3-5.1, below, illustrates the computation for estimating the effects of using an EOQ table, on the average inventory investment and the number of orders per year. It is assumed here that in using a 50 cost ratio EOQ table per figure 1-3.5 three items with monthly requirements between $60 and $100 will be ordered in a four-month supply and two items with monthly requirements between $240 and $400 will have a two-month supply. If the three items have a total monthly requirement of $225, ordering them in a four-month supply will result in a total EOQ value of $900 and the resulting order frequency of three per item will create nine orders during the year. Similarly if the two items have a total monthly requirement of $600, ordering them in a two-month supply will result in a total EOQ value of $1,200 and the resulting order frequency of six per item will create 12 orders during the year. The resulting number of orders per year for all five items will be 21. The "average" inventory investment is taken as one-half the total EOQ value or $1,050 since the value of the inventory investment will fluctuate anywhere from 0 to $2,100. While only two ranges of monthly requirements values and five items were used in the illustration of figure 3-5.1, the computation would be the same for any number of requirements ranges and any number of items. The same computational procedure could be used to estimate the expected average inventory investment (excluding safety stock) and total number of orders for EOQ procedures using formulas or other EOQ devices. The effects of a procedure using an EOQ formula would not be expected to differ substantially from an EOQ table procedure.

*Note: As discussed in par. 1 e of chapter 1, EOQ pertains only to the operating level. Effects of any given safety stock procedure would have to be estimated separately. In estimating holding cost per dollar of inventory (see par. 1 b of chapter 2), however, there is no need to differentiate between operating and safety stock since the cost to hold should be the same for both.

Range of Monthly Requirements	EOQ Months of Supply	Number of Items	Total Monthly Requirements	EOQ Value	EOQ Order Frequency	EOQ Order per Year
$ 60 to $100	4	3	$225	$ 900	3	9
240 to 400	2	2	600	1,200	6	12
				$2,100		21

Average Inventory Investment = $\frac{\$2,100}{2}$ = $1,050

Figure 3-5.1. Estimating Effects of EOQ

b. Estimating by Formula. An estimate of EOQ effects on average inventory investment (excluding safety stock) and total number of orders can also be computed by two formulas. Both formulas in figure 3-5.2, below, are simply adaptations of the basic EOQ formula $Q = C\sqrt{Y}$. These formulas can be used whenever the sum of the square roots of the inventory items is known or can be estimated. Such an estimate may be obtained by grouping items by requirements value range, finding the square root of the mid-point of the range, multiplying by the number of items in the range, and summing for all ranges. The square root of each item's requirements usually has been computed when a procedure using an EOQ formula is being followed. The estimates of average inventory investment and number of orders obtained from the formulas in figure 3-5.2 will not be as accurate as the method in figure 3-5.1 since the formulas do not limit the order size according to the prevailing policy. However, the formulas require considerably less work, once the sum of the square roots has been computed.

Formula for estimating average inventory investment (excluding safety stock).

$$\frac{\sum Q}{2} = \frac{C\sum\sqrt{Y}}{2}$$

Where Q = order quantity for one item

$\sum Q$ = sum of order quantities for all items

$\frac{\sum Q}{2}$ = average inventory investment for all items

C = cost factor based on $\sqrt{2\frac{P}{I}}$

$\sqrt{Y}$ = square root of annual requirements value for one item

$\sum\sqrt{Y}$ = sum of the square roots of annual requirements values for all items

Illustration of use where $\sum\sqrt{Y} = 210$ and $C = 10$

$$\frac{\sum Q}{2} = \frac{10 \times 210}{2} = \frac{2{,}100}{2}$$

$$\frac{\sum Q}{2} = \$1{,}050$$

Formula for total number of orders per year.

$$\sum F = \frac{\sum\sqrt{Y}}{C}$$

Where F = order frequency for one item

$\sum F$ = total number of orders per year for all items

Illustration of use where $\sum\sqrt{Y} = 210$ and $C = 10$

$$\sum F = \frac{210}{10}$$

$$\sum F = 21$$

Figure 3-5.2 Estimating Effects of EOQ by Formula

c. Use of Estimated Effects. The expected impact of an ordering system must be carefully evaluated if the system is to operate successfully. The expected inventory investment must be within the organization's budgetary limits and the expected number of orders per year must be within its order processing capability. After computing the expected average inventory investment and number of orders per year for an ordering procedure, the expected ordering and holding costs can also be computed. For example, the average inventory investment of $1,050 computed in figure 3-5.1 would result in a holding cost of $105 per year when computed at a cost rate of 10%, and the 21 orders per year would result in ordering costs of $105 when computed at a cost of $5 per order. The expected ordering and holding costs for a given ordering procedure may be compared with the actual costs of the current procedure or the expected costs for any other proposed procedure in order to evaluate the alternatives in cost terms before implementing any procedure.

6. MODIFICATIONS OF EOQ. A basic EOQ procedure is one which is based on the best available estimates of the costs for ordering and holding stock. Although basic EOQ is designed to reduce the total costs for ordering and holding to a minimum, there are times when the implementation of a basic EOQ procedure is not feasible or when a deviation from the basic EOQ procedure in use is necessary. In estimating the effects of an EOQ procedure before actual implementation it may be determined that the inventory investment or space requirement is greater than current limitation, or that the number of orders exceeds present order processing capabilities. In some circumstances a sudden shortage of funds or manpower may require that order quantities or frequencies be reduced from those prescribed by the basic EOQ procedure. (The limitation on inventory would, of course, restrict the safety stock level as well as the operating level determined by EOQ.) Whenever it is necessary to deviate from basic EOQ a "modified" EOQ procedure should be used until such time as basic EOQ may be implemented. Under modified EOQ, instead of using a cost factor or cost ratio based on cost estimates for ordering and holding stock, one that will provide the more expedient inventory investment or number of orders is used.

a. Use of Cost Factors. Figure 3-6.1, below, illustrates the use of cost factors that modify the effects of basic EOQ determined in figure 3-5.2. Using a cost factor of seven instead of 10 will result in an average inventory investment of $735 instead of $1,050. Using a cost factor of 15 instead of 10 results in 14 orders per year instead of 21.

Formula for modified average inventory investment (excluding safety stock):

$$\frac{\sum Q}{2} = \frac{C' \sum \sqrt{Y}}{2}$$

Where C' = cost factor for desired modification

Illustration of use where $\sum \sqrt{Y} = 210$ and $C' = 7$

$$\frac{\sum Q}{2} = \frac{7 \times 210}{2} = \frac{1,470}{2}$$

$$\frac{\sum Q}{2} = \$735$$

Formula for modified number of orders per year:

$$\sum F = \frac{\sum \sqrt{Y}}{C'}$$

Where C' = cost factor for desired modification

Illustration of use where $\sum \sqrt{Y} = 210$ and $C' = 15$

$$\sum F = \frac{210}{15}$$

$$\sum F = 14$$

Figure 3-6.1. Estimating Effects of Modified EOQ by Formula

b. Changes in Orders and Inventory. The cost factor which reduces the average inventory investment will at the same time increase the number of orders per year. Conversely the cost factor which reduces the number of orders per year will increase the average inventory investment. For example, the cost factor of seven which reduces the average inventory investment to $735 in figure 3-6.1 will also increase the number of orders from 21 to 30 $\left(\sum F = \frac{210}{7} = 30\right)$. The cost factor of 15 which reduces the number of orders to 14 in figure 3-6.1 will also increase the average inventory investment from $1,050 to $1,575.

$$\left(\sum \frac{Q}{2} = \frac{15 \times 210}{2} = \$1,575\right)$$

(1) A cost factor which is 70% of the basic EOQ cost factor (a change of - 30%) will result in an average inventory investment which is 70% of the basic EOQ investment (a change of - 30%) and a number of orders per year which is 1/70% or 143% of the basic EOQ number of orders (a change of + 43%). This again illustrates the inverse relationship between order quantity and number of orders. It also means that in order to obtain 70% of the investment, 70% of the cost factor should be used and to obtain 143% of the orders, 70% (1/1.43 = .70) of the cost factor should be used.

(2) Figure 3-6.2, below, lists some of the percentage changes in average inventory and number of orders that would result from given percentage changes in the cost factor. For example, a 25% increase in an EOQ cost factor will increase the average inventory investment by 25% and decrease the number of orders per year by 20%. This table can be used as a guide for determining how much to change the cost factor for given changes in investment or orders. The percentage changes in cost factor shown in figure 3-6.2 are not necessarily changes from the basic cost factor (i.e., the cost factor based on cost estimates to order and hold stock). The percentage changes relate to any cost factor for which the expected average inventory investment and number of orders per year have been estimated.

% Change in Cost Factor	% Change in Average Inventory Investment	% Change in Number of Orders per Year
+ 100	+ 100	- 50
+ 66	+ 66	- 40
+ 50	+ 50	- 33
+ 33	+ 33	- 25
+ 25	+ 25	- 20
+ 11	+ 11	- 10
0	0	0
- 10	- 10	+ 11
- 20	- 20	+ 25
- 25	- 25	+ 33
- 30	- 30	+ 43
- 33	- 33	+ 50
- 40	- 40	+ 66
- 50	- 50	+ 100

Figure 3-6.2. Effects of Cost Factor Changes on Inventory (Excluding Safety Stock) and Orders

c. Finding Cost Factor for Modification. It is also possible to find directly, the cost factor that will result in a desired average inventory investment (excluding safety stock) or number of orders per year, provided the sum of the square roots of the item requirements values has been computed. Adaptations of the formulas in figure 3-6.1, for this purpose, are shown in figure 3-6.3, below. These formulas may be used to find the cost factors for a modified EOQ without having a previously computed basic or modified cost factor. As illustrated, the cost factor that will result in an average inventory investment of $735 is 7, when the sum of the square roots of each item's requirements is 210. Similarly the cost factor that will result in 14 orders per year is 15.

Formula for cost factor to modify average inventory investment (excluding safety stock):

$$C' = \frac{\sum Q/2}{\sum \sqrt{Y/2}}$$

Illustration of use where $\sum \sqrt{Y} = 210$ and the desired average inventory investment is $\frac{\sum Q}{2} = \$735$:

$$C' = \frac{735}{210/2} = \frac{735}{105}$$

$$C' = 7$$

Formula for cost factor to modify number of orders per year:

$$C' = \frac{\sum \sqrt{Y}}{\sum F}$$

Illustration of use where $\sum \sqrt{Y} = 210$ and the desired number of orders per year, $\sum F = 14$:

$$C' = \frac{210}{14}$$

$$C' = 15$$

Figure 3-6.3. Finding Cost Factors for Modified EOQ

d. Use of Cost Ratios. The changes in cost ratio necessary to effect given changes in investment and orders are different from the changes in cost factor that would give the same results. Figure 3-6.4, below, lists some of the percentage changes in inventory and orders that would result from given percentage changes in the cost ratio. For example, a 56% increase in an EOQ cost ratio will increase the average inventory investment by 25% and decrease the number of orders per year by 20%. This table may be used as a guide for determining how much to change the cost ratio for given changes in investment or orders.

% Change in Cost Ratio	% Change in Average Inventory Investment	% Change in Number of Orders per Year
+ 300	+ 100	- 50
+ 175	+ 66	- 40
+ 125	+ 50	- 33
+ 77	+ 33	- 25
+ 56	+ 25	- 20
+ 23	+ 11	- 10
0	0	0
- 19	- 10	+ 11
- 36	- 20	+ 25
- 44	- 25	+ 33
- 51	- 30	+ 43
- 56	- 33	+ 50
- 64	- 40	+ 66
- 75	- 50	+ 100

Figure 3-6.4. Effects of Cost Ratio Changes on Inventory (Excluding Safety Stock) and Orders

(1) The cost factor used for modified EOQ will also provide the cost ratio upon which to construct a modified EOQ table. Figure 3-6.5, below, shows the relationship between the cost factor and the cost ratio. One may be obtained from the other by use of these simple formulas. This same relationship exists whether the cost ratio or cost factor is based on the actual estimates of costs to order and hold stock or has been modified to produce a particular result. Since cost factors may be converted to cost ratios, the methods described above for finding or changing cost factors for modified EOQ are also applicable to cost ratios.

Formula for obtaining cost factor from cost ratio:

$$C = \sqrt{2K}$$

Where C = Cost factor

K = Cost ratio based on $\frac{P}{I}$

Illustration of use where K = 50

$$C = \sqrt{2 \times 50} = \sqrt{100}$$

$$C = 10$$

Formula for obtaining cost ratio from cost factor:

$$K = \frac{C^2}{2}$$

Illustration of use where C = 10

$$K = \frac{100}{2}$$

$$K = 50$$

Figure 3-6.5. Relationship Between Cost Factor and Cost Ratio

(2) It is also possible to find directly the cost ratio that will result in the desired average inventory investment or number of orders per year. In order to determine these cost ratios, the sum of the square roots of the item requirements values must have been computed. Adaptations of the formulas in figure 3-6.3 to obtain cost ratios are shown in figure 3-6.6, below. These formulas may be used to find the cost ratios for a modified EOQ without having a previously computed basic or modified cost ratio. For example, the cost ratio that will result in an average inventory investment of $735 is 24.5, when the sum of the square roots of each item's requirements value is 210.

Formula for cost ratio to modify average inventory investment (excluding safety stock):

$$K' = 2\left(\frac{\sum Q/2}{\sum\sqrt{Y}}\right)^2$$

Where K' = Cost ratio for desired modification

Illustration of use where $\sum\sqrt{Y} = 210$ and the desired average inventory investment is $\frac{\sum Q}{2} = \$735$:

$$K' = 2\left(\frac{735}{210}\right)^2 = 2\left(3.5\right)^2 = 2 \text{ X } 12.25$$

$$K' = 24.5$$

Formula for cost ratio to modify number of orders per year:

$$K' = \frac{1}{2}\left(\frac{\sum\sqrt{Y}}{\sum F}\right)^2$$

Illustration of use where $\sum\sqrt{Y} = 210$ and the desired number of orders per year is $\sum F = 14$:

$$K' = \frac{1}{2}\left(\frac{210}{14}\right)^2 = \frac{1}{2}\,(15)^2 = \frac{1}{2} \text{ X } 225$$

$$K' = 112.5$$

Figure 3-6.6. Finding Cost Ratios for Modified EOQ

e. Table Alteration. Figures 3-6.4 and 3-6.6 make it possible to modify EOQ by either changing the cost ratio or finding the cost ratio that will result in the desired modification. Either method will permit the construction of an EOQ table based on the cost ratio that will achieve the desired modification. It is also possible to alter an existing EOQ table in order to achieve a desired modification. A comparison of the 50 cost ratio table in figure 3-3.3 and the 100 cost ratio table in figure 3-3.4 will indicate what is involved in altering an EOQ table. Changing from a 100 cost ratio table to a 50 cost ratio table would mean a decrease of 50% in the cost ratio. According to figure 3-6.4 a decrease of 51% in the cost ratio will result in a 30% reduction in the average inventory investment and a 43% increase in the number of orders per year. Approximately the same changes in investment and orders are expected by changing from figure 3-3.4 to figure 3-3.3. In order to accomplish these changes, therefore, the requirements values in each of the ranges for a given EOQ value, have been cut in half.

(1) Figure 3-6.7, below, lists some of the percentage changes in inventory and orders that would result from given percentage changes in the requirements values of an EOQ table. For example, a 56% increase in the requirements values of an EOQ table will increase the average inventory investment by 25% and decrease the number of orders per year by 20%. This table is the same as figure 3-6.4 except for the "% Change in Cost Ratio" which is now "% Change in Requirements Value."

% Change in Requirements Value	% Change in Average Inventory Investment	% Change in Number of Orders per Year
+ 300	+ 100	- 50
+ 175	+ 66	- 40
+ 125	+ 50	- 33
+ 77	+ 33	- 25
+ 56	+ 25	- 20
+ 23	+ 11	- 10
0	0	0
- 19	- 10	+ 11
- 36	- 20	+ 25
- 44	- 25	+ 33
- 51	- 30	+ 43
- 56	- 33	+ 50
- 64	- 40	+ 66
- 75	- 50	+ 100

Figure 3-6.7. Effects of Changes in Requirements Values in EOQ Table on Inventory (Excluding Safety Stock) and Orders

(2) Figure 3-6.8, below, illustrates how the requirements values of a cost ratio 50 table are changed in order to reduce inventory. In this case, in order to reduce inventory by 30% the requirements values are reduced by 51% (actually rounded to 50%).

Monthly Requirements	Modified Ranges (50% of Monthly Requirements)	EOQ in Months of Supply
$ 10 and less	$ 5 and less	12
10 to $ 20	5 to $ 10	9
20 to 40	10 to 20	6
40 to 60	20 to 30	5
60 to 100	30 to 50	4
100 to 160	50 to 80	3
160 to 240	80 to 120	$2\frac{1}{2}$
240 to 400	120 to 200	2
400 to 800	200 to 400	$1\frac{1}{2}$
800 and over	400 and over	1

Figure 3-6.8. Changing EOQ Table for Modified EOQ

f. Errors in Modified EOQ Table. Due to the inherent error in an EOQ table the desired effects of a modified EOQ table are seldom obtained exactly. Figure 3-6.9 estimates the effects of a modified EOQ table designed to reduce the average inventory investment determined in figure 3-5.1 by 30%. It is assumed that the 3 items in the $60 to $100 range of the cost ratio 50 table will fall in the $50 to $80 range of the modified table and the 2 items in the $240 to $400 range will fall in the $200 to $400 range. The desired modification was a 30% reduction of the $1,050 investment to $735. As it turned out,

however, the investment was reduced to $787.50, which is only a 25% reduction from $1,050. This inaccuracy essentially may be ascribed to the EOQ table method whereby a single value arbitrarily represents the EOQ values for items within a range of monthly requirements values.

Range of Monthly Requirements	EOQ Months of Supply	Number of Items	Total Monthly Requirement	EOQ Value	EOQ Order Frequency	EOQ Orders per Year
$ 50 to $ 80	3	3	$225	$ 675	4	12
200 to 400	1½	2	600	900	8	16
				$1,575		28

Average Inventory Investment = $\frac{\$1,575}{2}$ = $787.50

Figure 3-6.9. Estimating Effects of Modified EOQ Table

g. Changes in Costs. After computing the expected average inventory investment and the number of orders per year for a modified EOQ procedure, the expected ordering and holding costs can also be computed. For example, the average inventory investment of $787.50 computed in figure 3-6.9 would result in a holding cost of $78.75 when computed at a rate of 10% of the average inventory investment, and the 28 orders per year would result in ordering costs of $140 when computed at $5 per order. The total costs for the modified EOQ therefore would be $218.75 as compared to $210 for the basic EOQ. Although the modified EOQ does not reduce the total costs for ordering and holding stock to the minimum, as basic EOQ does, for any desired inventory investment it will reduce the number of orders and resulting ordering costs to the minimum, or for any desired number of orders it will reduce the inventory investment and resulting holding costs to the minimum.

7. THE ECONOMIC PURCHASE QUANTITY. A basic EOQ procedure will result in the lowest total cost for ordering and holding stock by considering the cost per order, holding cost per dollar of inventory, and the dollar value of requirements. While ordering and holding costs are the most important costs to consider when developing a replenishment procedure which will minimize costs, there are

times when other costs must also be considered. Two other costs could have an effect in determining the most economical purchase quantity. First there is the unit price discount for quantity purchases, and secondly, the discount rates for transportation of given quantities. The net effect of each of these is to reduce the unit price of the item being purchased and thereby reduce the purchase cost over the year. However, since this cost reduction is usually contingent on purchasing a quantity other than the basic EOQ, it must be evaluated in light of the increased costs which will result from deviating from basic EOQ. The same type of practical and policy limit considered necessary for EOQ is also necessary for EPQ. For example, the upper and lower limits for purchase quantities, in months of supply and dollar values should be the same as for EOQ. Similarly, caution must be exercised in ordering shelf-life items, or items which are bulky or difficult to store.

a. Evaluation of Costs. Figure 3-7.1, below, illustrates the evaluation of costs in order to obtain the economic purchase quantity. The 2% discount would reduce the unit price from $.25 to $.245 thereby saving $18 ($900 - $882) per year on the purchase of the annual requirement. Use of the discount, however, would require a greater order quantity than prescribed by EOQ thereby increasing the total costs for ordering and holding from $30.00 to $32.05. In this example the increased total costs for ordering and holding are not sufficient to offset the saving in the annual purchase cost of requirements and the discount should be taken. It will be noted that the increased order quantity causes the average inventory and holding costs to increase but also causes the number of orders per year and ordering costs to decrease. The net effect whenever there is a deviation from basic EOQ is to increase the total of the ordering and holding costs.

A. Basic EOQ where Cost to Order (P) = \$5, Cost to Hold (I) = 10%;

Annual Requirements in Units (Y_u) = 3,600; Unit Price (V) = \$.25

$$Q_u = C\sqrt{\frac{Y_u}{V}} = 10\sqrt{\frac{3,600}{.25}} = 10 \text{ X } 120 = 1,200 \text{ units}$$

Q = 1,200 X \$.25 = \$300

Annual Costs:

Purchase Cost of Annual Requirements

Y_u X V = 3,600 X \$.25 = \$900.00

Holding Cost

$Q_u/2$ X V X I = 1,200/2 X \$.25 X 10% = 15.00

Ordering Cost

Y_u/Q_u X P = 3,600/1,200 X \$5 = 15.00

Total Costs \$930.00

B. EPQ where 2% discount is offered for Purchase Quantity of 1,800 units (1,800 X \$.245 = \$441)

Annual Costs:

Purchase Cost of Annual Requirements

Y_u X V = 3,600 X \$.245 = \$882.00

Holding Cost

$Q_u/2$ X V X I = 1,800/2 X \$.245 X 10% = 22.05

Ordering Cost

Y_u/Q_u X P = 3,600 X 1,800 X \$5 = 10.00

Total Costs \$914.05

Figure 3-7.1. Evaluation of Economic Purchase Quantity (EPQ)

(1) Figure 3-7.2, below, which is based on the last 2 columns in figure 2-3.2 shows the expected percentage increases in the total costs for ordering and holding when the order quantities change from basic EOQ. This table indicates that the total cost is increased by 8% when the order quantity is 50% greater than basic EOQ. In figure 3-7.1 the order quantity was 50% greater than basic EOQ (1800 compared to 1200) but the increase in total costs was only 7% ($32.05 compared to $30.00). The reason for this discrepancy is that the holding cost for the discount quantity in figure 3-7.1 was computed on the basis of the discounted unit price. This discrepancy is minor and should not detract from the usefulness of figure 3-7.2 in determining whether or not purchase cost savings will be offset by the increase in the total costs for ordering and holding.

Percent Change from EOQ	Percent Change in Total Costs
- 40	+ 13
- 25	+ 4
0	0
+ 25	+ 2.5
+ 50	+ 8
+ 75	+ 16
+ 100	+ 25
+ 125	+ 35
+ 150	+ 45
+ 200	+ 67
+ 250	+ 89
+ 300	+ 112
+ 350	+ 136
+ 400	+ 160
+ 500	+ 208
+ 700	+ 306
+1100	+ 504

Figure 3-7.2. Expected Changes in Total Costs of Ordering and Holding for Changes from Basic EOQ

(2) In using figure 3-7.2 to evaluate the economic purchase quantity, it also would be useful to be able to estimate in simpler fashion, the total costs of ordering and holding under basic EOQ. Figure 3-7.3, below, illustrates a one-step computation for the total costs for ordering and holding under basic EOQ. This method is based on holding costs being equal to ordering costs when basic EOQ is used.

Formula:

$$\text{Total Costs} = Q_u \, VI$$

Illustration of Use where Q_u = 1,200; V = \$.25, I = 10%

$$\text{Total Costs} = 1{,}200 \times \$.25 \times 10\%$$

$$\text{Total Cost} = \$30$$

Figure 3-7.3. Estimating Total Costs for Basic EOQ

(3) As a practical matter, the method of evaluating EPQ as shown in figure 3-7.1 may be used whenever a discount opportunity must be considered. Under this method a work-sheet format may be developed which would indicate the computational steps and the decision to accept or decline the discount. This procedure may also make use of the method for estimating basic EOQ total costs as shown in figure 3-7.2 and the method for estimating changes in total costs when changing from EOQ, as shown in figure 3-7.3. The method in figure 3-7.1 may also be used for evaluating EPQ when there are several different discounts offered based on purchase quantity. The evaluation in that case would include comparisons for each additional price break.

b. EPQ Table. Where the volume of EPQ evaluations is large enough to warrant it, a previously computed table, nomograph, slide rule, or chart may be computed which quickly indicates the EPQ decision for a given set of ordering and holding costs, requirements values and discount rates. Figure 3-7.4, below, is an EPQ table which can serve as a guide when the costs

are $5 to order and 10% to hold, and the price discounts are .5%, 1%, 1.5%, or 2%. For selected EOQ dollar values, this table shows the largest economic purchase quantity dollar values for given discounts. For example, when the EOQ is $300, the largest EPQ for a 2% price discount is $857.

Annual Requirements	EOQ	Limits of EPQ for Given Discounts			
		.5%	1.0%	1.5%	2.0%
$ 100	$ 100	$ 138	$ 157	$ 173	$ 223
182	135	195	227	254	279
400	200	313	374	428	478
576	240	391	475	550	619
900	300	517	641	752	857
1,600	400	747	954	1,142	1,321
2,304	480	949	1,236	1,500	1,750
3,600	600	1,280	1,712	2,112	2,500
6,400	800	1,906	2,640	3,331	4,003
$14,400	$1,200	$3,421	$4,995	$6,502	$7,983

Figure 3-7.4. Economic Purchase Quantity Table ($5 to order and 10% to hold)

(1) Figure 3-7.5, below, is in effect a continuation of the EPQ evaluation in figure 3-7.1. As indicated the total costs are evaluated for an EPQ of 3,498 units valued at $857 (at discount price), and the EPQ of 4,000 units valued at $980 (at discount price). The total cost for purchasing 3,498 units ($857) is $930 which is exactly the total cost for purchasing the basic EOQ quantity. The purchase of a larger quantity results in a greater total cost. A purchase quantity of 4,000 units ($980) results in a total cost of $936.55.

Figure 3-7.4 shows the EPQ dollar values for given discount rates that will result in the same total cost as the corresponding EOQ dollar value. These EPQ values are therefore the upper limits of the economic purchase quantities. Purchase quantities below these values are economical while those above are not economical. Figure 3-7.4 is simply a guide for making EPQ decisions relative to quantities greater than basic EOQ. It may be extended for EOQ values below $100 and above $1,200 or may be refined for intervening EOQ values.

A. EPQ where 2% discount is offered for purchase quantity of 3,498 units (3,498 X $.245 = $857); Cost to Order (P) = $5; Cost to Hold (I) = 10%; Annual Requirement in Units (Y_u) = 3,600; Unit Price (V) = $.25

Annual Costs:

Purchase Cost of Requirements

Y_u X V = 3,600 X $.245 = $882.00

Holding Cost

$Q_u/2$ X V X I = 3,498/2 X $.245 X 10% = 42.85

Ordering Cost

Y_u/Q_u X P = 3,600/3,498 X $5 = 5.15

Total Costs $930.00

B. EPQ when 2% discount offered for purchase quantity of 4,000 units (4,000 X $.245 = $980)

Annual Costs:

Purchase Cost of Requirements

Y_u X V = 3,600 X $.245 = $882.00

Holding Costs

$Q_u/2$ X V X I = 4,000/2 X $.245 X 10% = 49.00

Ordering Cost

Y_u/Q_u X P = 4,000/3,600 X $5 = 5.55

Total Costs $936.55

Figure 3-7.5. Evaluation of EPQ Limits

(2) Figure 3-7.6, below, gives the formula for determining EPQ limits and illustrates its use in computing one of the EPQ limits given in figure 3-7.4. Since this formula is not simple to use in manual computations, its use is not encouraged for individual EPQ decisions or for constructing a table of EPQ limits that would receive infrequent use. The formula in figure 3-7.6 or variations of it does lend itself to other computational devices such as charts, nomographs, and slide rules.

Formula:

$$X = \frac{DY + QI + \sqrt{(DY + QI)^2 - 2\,PIY\,(1-D)}}{I}$$

Where X = EPQ dollar value limit

D = Percentage price discount

Y = Dollar value of annual requirements

Q = EOQ in dollar value

I = Cost to hold

P = Cost to order

Illustration of Use where D = 2%; Y = \$900; Q = \$300

I = 10%; P = \$5

$$X = \frac{.02X\$900 + \$300X.10 + \sqrt{(.02X\$900 + \$300X.10)^2 - 2X\$5X.10X\$900(1-.02)}}{.10}$$

$$X = \frac{48 + \sqrt{(48)^2 - 882}}{.10} = \frac{48 + \sqrt{2,304 - 882}}{.10} = \frac{48 + \sqrt{1,422}}{.10} = \frac{48 + 37.7}{.10}$$

X = \$857

Figure 3-7.6. Computation of EPQ Limits

This appendix offers a simple step-by-step procedure for applying EOQ. The procedure may be followed wholly or in part if it is desired to use an EOQ table. The procedure contains instructions for:

1. the development of a cost ratio for the EOQ table
2. construction of an EOQ table
3. determining effects of EOQ
4. modifications of an EOQ table

1. DEVELOPMENT OF COST RATIO FOR EOQ TABLE.

a. Ordering Costs. For each group of stocked items involving the same ordering procedure, list the estimated annual costs (including accounting, ADP, and other applicable costs) for the following elements:

(1) Reviewing the item _____

(2) Preparing and processing the requisition or purchase request _____

(3) Selection of a supplier (includes preparing and issuing price inquiries and receiving, tabulating, and evaluating quotations). _____

(4) Preparing and processing the purchase order _____

(5) Preparing and processing receiving reports _____

(6) Receiving, inspecting, and storing stock

(7) Posting receipts and stock records _____

(8) Preparing and processing payments _____

(9) Other ordering costs _____

Total Annual Ordering Costs _____

b. Cost for Ordering an Item. Divide the Total Annual Ordering Costs obtained in 1 a by the total number of times these items were ordered during the year.

Cost for Ordering an Item =

$$\frac{\text{Total Annual Ordering Costs}}{\text{Number of Times Items were Ordered during Year}}$$

c. Holding Costs. For each group of stocked items involving the same storage and warehousing procedures, list the estimated annual costs (including accounting, ADP, and other applicable costs) for the following elements:

(1) Taking physical inventory _____

(2) Preparing and processing inventory adjustments _____

(3) Prevention of deterioration _____

(4) Repacking and rewarehousing _____

(5) Storage space (if applicable) _____

(6) Interest on average annual inventory investment (at 4 1/2% per year) _____

(7) Annual inventory losses for excess, obsolescence, deterioration, loss, theft, damage _____

(8) Other holding costs _____

Total Annual Holding Costs _____

d. Holding Cost per Dollar of Inventory. Divide the Total Annual Holding Cost obtained in 1 c by the average dollar value of inventory held for these items during the year and multiply by 100 to express as a percentage.

Holding Cost Per Dollar of Inventory =

$$\frac{\text{Total Annual Holding Costs}}{\text{Average Annual Inventory Dollar Value}} \times 100$$

e. Cost Ratio. Divide the Cost for Ordering an Item obtained in 1 b by the Holding Cost Per Dollar of Inventory obtained in 1 d for each group of items that has essentially the same costs to order and to hold.

$$\text{Cost Ratio} = \frac{\text{Cost for Ordering an Item}}{\text{Holding Cost Per Dollar of Inventory}}$$

2. CONSTRUCTION OF EOQ TABLE.

Format #1

(1) Ranges of Basic Values	(2) Ranges of Monthly Requirements in Dollars (Cost Ratio X Col. 1)	(3) EOQ Months of Supply
.20 and less		12
.20 to .40		9
.40 to .80		6
.80 to 1.20		5
1.20 to 2.00		4
2.00 to 3.20		3
3.20 to 4.80		2 1/2
4.80 to 8.00		2
8.00 to 16.00		1 1/2
16.00 and over		1

The EOQ table is constructed by completing Format #1 as follows:

a. In column 2, compute the Ranges of Monthly Requirements in Dollars by multiplying the Cost Ratio obtained in 1 e, above, by each of the Ranges of Basic Values (column 1).

b. After completing Format #1, the EOQ table will consist of columns 2 and 3.

3. PREDETERMINING EFFECTS OF EOQ.

a. Total Number of Orders per Year and Total Ordering Costs.

Format #2

(1) Ranges of Monthly Requirements in Dollars	(2) EOQ Months of Supply	(3) Order Frequency	(4) Number of Items	(5) Number of Orders per year (Col. 3 X Col. 4)
	12	1		
	9	1.3		
	6	2		
	5	2.4		
	4	3		
	3	4		
	2 1/2	4.8		
	2	6		
	1 1/2	8		
	1	12		
		Total		

(1) The Total Number of Orders per Year is obtained by completing Format #2 as follows:

(a) In column 1, insert the same Ranges of Monthly Requirements in Dollars computed in column 2 of Format #1.

(b) In column 4, show the Number of Items falling within each of the Ranges of Monthly Requirements in Dollars (column 1).

(c) In column 5, compute the Number of Orders per Year by multiplying each Order Frequency in column 3

by the Number of Items on the corresponding line in column 4.

(d) The Total Number of Orders per Year is the total of column 5.

(2) The total Ordering Costs is obtained by multiplying the Total Number of Orders per Year obtained in 3 a (1)(d) by the Cost for Ordering an Item obtained in 1 b.

Total Ordering Costs =

Total Number of Orders per Year X

Cost for Ordering an Item

b. Average Inventory Investment and Total Holding Costs.

Format #3

(1) Ranges of Monthly Requirements in Dollars	(2) EOQ Months of Supply	(3) Dollar Value of Monthly Requirements	(4) Dollar Value of Order Quantities (Col. 2 X Col. 3)
	12		
	9		
	6		
	5		
	4		
	3		
	2 1/2		
	2		
	1 1/2		
	1		
	Total		
Total	÷ 2		

(1) The Average Inventory Investment is obtained by completing Format #3 as follows:

(a) In column 1, insert the same Ranges of Monthly Requirements in Dollars computed in column 2 of Format #1.

(b) In column 3, show the Dollar Value of Monthly Requirements for items falling within each of the ranges in column 1.

(c) In column 4, compute the Dollar Value of Order Quantities by multiplying each of the EOQ Months of Supply in column 2 by the Dollar Value of Monthly Requirements on the corresponding line in column 3.

(d) The total of column 4, Dollar Value of Order Quantities divided by 2 is the Average Inventory Investment.

Note: Average Inventory Investment does not include safety stock. If required for comparison with actual operations or otherwise, an estimate of safety stock investment will have to be added.

(2) The Total Holding Costs is obtained by multiplying the Average Inventory Investment obtained in 3 b (1)(d) by the Holding Cost per Dollar of Inventory obtained in 1 d.

Total Holding Costs =

Average Inventory Investment

X Holding Cost Per Dollar of Inventory

4. MODIFICATIONS OF EOQ TABLE.

Format #4

(1) Original Ranges of Monthly Requirements in Dollars	(2) Modified Ranges of Monthly Requirements in Dollars (Col. 1 X Change Factor)	(3) EOQ Months of Supply
		12
		9
		6
		5
		4
		3
		2 1/2
		2
		1 1/2
		1

a. The EOQ table is modified by completing Format #4 as follows:

(1) In column 1, insert the same Ranges of Monthly Requirements in Dollars computed in column 2 of Format #1.

(2) In column 2, compute the Modified Ranges of Monthly Requirements in Dollars by multiplying each of the ranges in column 1 by the appropriate Change Factor for Requirements Values selected from column 2 of Table 1 or Table 2.

(3) After completing Format #4, the modified EOQ table will consist of columns 2 and 3.

b. Modification for Limiting the Total Number of Orders per Year.

TABLE 1

(1) Desired % Reduction in Total Number of Orders per Year	(2) Change Factor for Requirements Values	(3) Resulting % Increase in Average Inventory Investment*
- 10	1.23	+ 11
- 20	1.56	+ 25
- 25	1.77	+ 33
- 33	2.25	+ 50
- 40	2.75	+ 66
- 50	4.00	+100

(1) The selection of a Change Factor for Requirements Values (column 2 of table 1) is based on the Desired % Reduction in Total Number of Orders per Year (column 1 of table 1). For example, to reduce the Total Number of Orders per Year by 10%, a Change Factor of 1.23 is selected.

(2) The Resulting % Increase in Average Inventory Investment is shown in column 3 of table 1 for each Change Factor in column 2 of table 1.

*Does not include safety stock. The change factor has no effect on safety stock.

c. Modification for Limiting the Average Inventory Investment.

TABLE 2

(1) Desired % Reduction in Average Inventory Investment*	(2) Change Factor for Requirements Values	(3) Resulting % Increase in Total Number of Orders per Year
- 10	.81	+ 11
- 20	.64	+ 25
- 25	.56	+ 33
- 30	.49	+ 43
- 33	.44	+ 50
- 40	.36	+ 66
- 50	.25	+100

(1) The selection of a Change Factor for Requirements Values (column 2 of table 2) is based on the Desired % Reduction in Average Inventory Investment (column 1 of table 2). For example, to reduce the Average Inventory Investment by 10% a Change Factor of .81 is selected.

(2) The Resulting % Increase in Total Number of Orders per Year is shown in column 3 of table 2.

*Does not include safety stock. The change factor has no effect on safety stock.

A	$\sqrt{A}$	A	$\sqrt{A}$	A	$\sqrt{A}$	A	$\sqrt{A}$	A	$\sqrt{A}$	A	$\sqrt{A}$
.01	.10	4.5	2.1	26	5.1	74	8.6	170	13.0	305	17.5
.02	.14	5.0	2.2	27	5.2	76	8.7	175	13.2	310	17.6
.03	.17	5.5	2.4	28	5.3	78	8.8	180	13.4	315	17.7
.04	.20	6.0	2.5	29	5.4	80	8.9	185	13.6	320	17.9
.05	.22	6.5	2.6	30	5.5	82	9.1	190	13.8	325	18.0
.06	.24	7.0	2.7	31	5.6	84	9.2	195	14.0	330	18.2
.07	.26	7.5	2.7	32	5.7	86	9.3	200	14.1	335	18.3
.08	.28	8.0	2.8	34	5.8	88	9.4	205	14.3	340	18.4
.09	.30	8.5	2.9	36	6.0	90	9.5	210	14.5	345	18.6
.1	.32	9.0	3.0	38	6.2	92	9.6	215	14.7	350	18.7
.2	.45	9.5	3.1	40	6.3	94	9.7	220	14.8	355	18.8
.4	.63	10.0	3.2	42	6.5	96	9.8	225	15.0	360	19.0
.6	.77	11	3.3	44	6.6	98	9.9	230	15.2	365	19.1
.8	.89	12	3.5	46	6.8	100	10	235	15.3	370	19.2
1.0	1.00	13	3.6	48	6.9	105	10.2	240	15.5	375	19.4
1.2	1.1	14	3.7	50	7.1	110	10.5	245	15.7	380	19.5
1.4	1.2	15	3.9	52	7.2	115	10.7	250	15.8	385	19.6
1.6	1.3	16	4.0	54	7.3	120	11.0	255	16.0	390	19.7
1.8	1.3	17	4.1	56	7.5	125	11.2	260	16.1	395	19.9
2.0	1.4	18	4.2	58	7.6	130	11.4	265	16.3	400	20.0
2.2	1.5	19	4.4	60	7.7	135	11.6	270	16.4	405	20.1
2.4	1.6	20	4.5	62	7.9	140	11.8	275	16.6	410	20.2
2.6	1.6	21	4.6	64	8.0	145	12.0	280	16.7	415	20.4
2.8	1.7	22	4.7	66	8.1	150	12.2	285	16.9	420	20.5
3.0	1.7	23	4.8	68	8.2	155	12.4	290	17.0	425	20.6
3.5	1.9	24	4.9	70	8.4	160	12.6	295	17.2	430	20.7
4.0	2.0	25	5.0	72	8.5	165	12.8	300	17.3	435	20.9

Appendix B. Square Root Values

A	$\sqrt{A}$	A	$\sqrt{A}$	A	$\sqrt{A}$	A	$\sqrt{A}$	A	$\sqrt{A}$	A	$\sqrt{A}$
440	21.0	575	24.0	750	27.4	3,100	55.8	6,250	79.1	26,000	161
445	21.1	580	24.1	775	27.8	3,200	56.6	6,500	80.6	27,000	164
450	21.2	585	24.2	800	28.3	3,300	57.4	6,750	82.2	28,000	167
455	21.3	590	24.3	850	29.1	3,400	58.3	7,000	83.7	29,000	170
460	21.4	595	24.4	900	30.0	3,500	59.2	7,250	85.1	30,000	173
465	21.6	600	24.5	950	30.8	3,600	60.0	7,500	86.6	32,500	180
470	21.7	605	24.6	1,000	31.6	3,700	60.8	7,750	88.0	35,000	187
475	21.8	610	24.7	1,100	33.2	3,800	61.6	8,000	89.4	37,500	193
480	21.9	615	24.8	1,200	34.6	3,900	62.4	8,500	92.2	40,000	200
485	22.0	620	24.9	1,300	36.1	4,000	63.2	9,000	94.9	42,500	206
490	22.1	625	25.0	1,400	37.4	4,100	64.0	9,500	97.5	45,000	212
495	22.2	630	25.1	1,500	38.7	4,200	64.8	10,000	100	47,500	218
500	22.4	635	25.2	1,600	40.0	4,300	65.8	11,000	105	50,000	224
505	22.5	640	25.3	1,700	41.2	4,400	66.3	12,000	110	52,500	229
510	22.6	645	25.4	1,800	42.4	4,500	67.1	13,000	114	55,000	235
515	22.7	650	25.5	1,900	43.6	4,600	67.8	14,000	118	57,500	240
520	22.8	655	25.6	2,000	44.7	4,700	68.6	15,000	122	60,000	245
525	22.9	660	25.7	2,100	45.8	4,800	69.3	16,000	126	62,500	250
530	23.0	665	25.8	2,200	47.0	4,900	70.0	17,000	130	65,000	255
535	23.1	670	25.9	2,300	48.0	5,000	70.7	18,000	134	67,500	260
540	23.2	675	26.0	2,400	49.0	5,100	71.4	19,000	138	70,000	265
545	23.3	680	26.1	2,500	50.0	5,200	72.1	20,000	141	75,000	274
550	23.5	685	26.2	2,600	51.0	5,300	72.8	21,000	145	80,000	283
555	23.6	690	26.3	2,700	52.0	5,400	73.5	22,000	148	85,000	291
560	23.7	695	26.4	2,800	52.9	5,500	74.2	23,000	152	90,000	300
565	23.8	700	26.5	2,900	53.9	5,750	75.8	24,000	155	95,000	308
570	23.9	725	26.9	3,000	54.8	6,000	77.5	25,000	158	100,000	316

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